Katie Heenan and Katie Stanovich – Algebraic Analysis of Mutually Orthogonal Latin Squares

The paper and the presentation are very good, apart from a few points that might have been explained more completely or carefully. See especially points 5,6,7 below. Writing about mathematics is different (and harder) than other sorts of writing because you have to be careful to state things so precisely.

Some Specific Comments

1) Section 2.1 is focused on *magic squares*. You do say, after the first paragraphs, that Latin squares are different. It might have been good to start with that, though, to make it clearer that the two concepts are different. On the other hand, they are also even more closely related than you indicate because Euler also wrote a paper where he used Latin squares to construct magic squares. One idea is as follows. For instance, if $n = 5$ and

$$L = \begin{pmatrix} a & b & c & d & e \\ b & c & d & e & a \\ c & d & e & a & b \\ d & e & a & b & c \\ e & a & b & c & d \end{pmatrix} \qquad \text{and} \qquad L' = \begin{pmatrix} E & D & C & B & A \\ A & E & D & C & B \\ B & A & E & D & C \\ C & B & A & E & D \\ D & C & B & A & E \end{pmatrix}$$

are any Latin square matrices with $a+b+c+d+e = 5e$ and $A+B+C+D+E = 5E$, and $\{a, b, c, d, e\} = \{1, 2, 3, 4, 5\}$, $\{A, B, C, D, E\} = \{0, 1, 2, 3, 4\}$, then L and L' are mutually orthogonal Latin squares (of the cyclic type you discuss on page 5), and moreover the linear combination $L + 5L'$ (using arithmetic in $\mathbf{Z}$) is a *magic square*. Something similar works for any odd n. (This is a nice exercise – try it!) Hope and Margot mentioned this connection in their talk. Euler also discussed different constructions, and not every magic square can be found this way, but it's a nice relation between these two different types of interesting square arrays of numbers.
2) Page 4 – your *standard form* Latin squares use entries in $\{0, 1, \ldots, n-1\}$ but the definition of a Latin square says specifically the entries are in $\{1, 2, \ldots, n\}$ so you are being slightly inconsistent.
3) Pages 7 and 8 – there is an extra i in the "Hall condition" used in the Marriage Theorem. The statement should be that a transversal of $U = (A_1, \ldots, A_n)$ exists if and only if

$$|\cup_{i \in I} A_i| \geq |I|$$

for all $I \subseteq [n]$. (This might have been my typo in the email I sent you, but it doesn't make sense as

$$|\cup_{i \in I} A_i i| \geq |I|$$

4) Page 9 – The multiplication table mod 5 is a Latin square of order $n = 4$. To get a Latin square of order $n = 5$ from $\mathbf{Z}_5$, you could take the *addition table*, though.

5) Step 1 of proof on Page 10 – Note that the thing that lets you conclude $i = i'$ from $k(i-1)+j-1+k(i'-1)+j-1$ is the fact that $k \not\equiv 0 \bmod n$. This means that k has a multiplicative inverse mod n (using the fact that n is prime). Hence

$$k(i-1) \equiv k(i'-1) \equiv \bmod n \Rightarrow i \equiv i' \bmod n.$$

6) Step 2 of the proof on Page 10 – there's no k left after you subtract this time, just $j-1 \equiv j'-1 \bmod n$.
7) Page 11 – Strictly speaking, if you subtract the 2 equations in the middle of the page you get

$$(k-k')(i-1) = (k-k')(i'-1)$$

Your equation comes by subtracting these to get

$$(k-k')(i-i') = 0.$$

The $= 0$ (maybe more precisely $\equiv 0 \bmod n$) is important, because that's how you are going to use the assumption that n is prime. From

$$(k-k')(i-i') \equiv 0 \bmod n$$

you get

$$k \equiv k' \bmod n \quad \text{or} \quad i \equiv i' \bmod n.$$

Final Project Grade Computation

Bibliography: 10/10

Paper: 55/60

Presentation: 30/30

Total: 95/100