College of the Holy Cross, Fall Semester, 2015
MATH 133, section 1, Midterm 4
Thursday, December 10

Your Name: _____

Instructions: Please show all work necessary to justify your answers, and write your answers in the spaces provided. Use the back of the preceding page if you need more space for scratch work. There are 100 possible points distributed as below.

Please do not write in the space below

Problem	Points/Poss
1	/ 25
2	/ 20
3	/ 30
4	/ 25
Total	/100

1. Find $\frac{dy}{dx}$; do not simplify:

A) (7.5) $y = \ln(\sin(x) + \cos(x))$

B) (7.5) $y = \tan^{-1}(e^{3x})$

C) (10) $x^2y^4 - 4\sin^{-1}(y^2) + x = 0$ (use implicit differentiation)

2. (20) A stationary observer watches a weather balloon being launched from a point 1000 feet away from her position. The balloon rises at a rate of 20 feet per second. How fast is the distance between the balloon and the observer changing when the balloon is 400 feet above the ground?

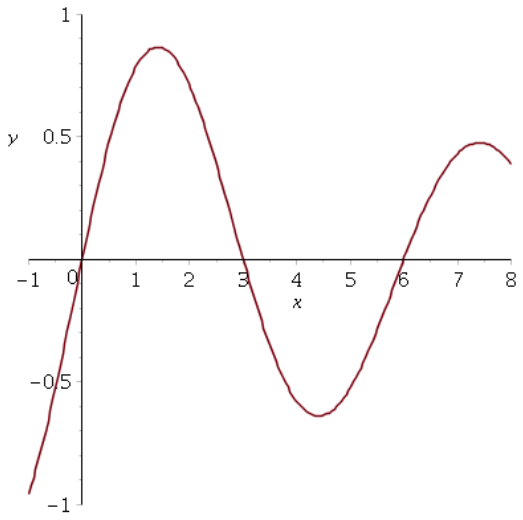

Figure 1: $y = f'(x)$ for Problem 3

3. All parts of this question refer to the plot in Figure 1, *which is* $y = f'(x)$ *for some function* $f(x)$. Assume the whole domain of the functions $f(x)$ and $f'(x)$ is the interval $[-1, 8]$ shown.

   (A) (10) Give approximate values for the critical points of $f(x)$ in the interval shown:
   Answer: _____

   (B) (5) Briefly, in your own words, state how the First Derivative Test distinguishes between local maxima, local minima, and critical points that are neither:

   (C) (5) Identify each of the points you found in part (A) as a local maximum, local minimum, or neither:
   Answer: _____

   (D) (5) Find approximate values for all the inflection points of $f(x)$.
   Answer: _____

   (E) (5) Over which intervals is $y = f(x)$ concave up? concave down?
   Concave up: _____ Concave down: _____

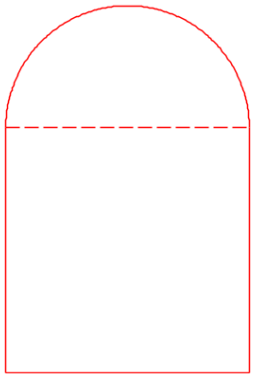

Figure 2: The window for Problem 4.

4. A church window has the shape of a rectangle surmounted by a semicircle (see Figure 2). The total perimeter of the window is 800 cm. What should the dimensions be to make the area of the window be as large as possible (so that it will admit the most light possible)? Useful information: The area of a circle of radius r is $A = \pi r^2$ and the circumference is $C = 2\pi r$. The area of a rectangle is the product of its length and width. The parts of this problem will lead you to the answer.

   (A) (5) Call the horizontal side of the rectangle x and the vertical side y. Express the total area of the window and the total perimeter in terms of x and y.

   (B) (5) Solve for y in terms of x using the perimeter equation and substitute into the area equation.

(C) (5) Determine a critical point of the area function.

(D) (5) How do you know your critical point is a maximum of the area?

(E) (5) What are the dimensions x and y of the window of maximum area?