MATH 133 – Calculus with Fundamentals 1
Solutions for Quiz 7 – November 19, 2015

Questions

1) (a) (7) Find $\frac{dy}{dx}$ by implicit differentiation given that $x^2y^3 - 5xy + x = 1$.

Solution: Using implicit differentiation means we have to think of y *as an implicitly defined function of* x. This means that the x^2y^3 and $-5xy$ terms are products of functions of x and must be differentiated by the product rule (and chain rule for the first):

$$x^2 \cdot 3y^2\frac{dy}{dx} + 2xy^3 - 5x\frac{dy}{dx} - 5y + 1 = 0.$$

Then we take the terms without $\frac{dy}{dx}$ to the other side:

$$x^2 \cdot 3y^2\frac{dy}{dx} - 5x\frac{dy}{dx} = 5y - 2xy^3 - 1,$$

Then factor out the $\frac{dy}{dx}$ on the left:

$$\left(3x^2y^2 - 5x\right)\frac{dy}{dx} = 5y - 2xy^3 - 1,$$

and finally divide by the $3x^2y^2 - 5x$ to solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = \frac{5y - 2xy^3 - 1}{3x^2y^2 - 5x}.$$

(b) (3) Find the equation of the tangent line to the curve with the equation $x^2y^3-5xy+x = 1$ at $(x, y) = (0, 1)$ using your answer from (a).

Solution: I goofed on this part. The point I meant was $(1, 0)$, not $(0, 1)$. As some of you noticed $\frac{dy}{dx}$ is not even defined at $(0, 1)$ (and even worse, that point doesn't satisfy the equation of the curve). So I gave everyone full credit for this part since it was my mistake. Here's the way it would work with the correct point $(x, y) = (1, 0)$: The slope is found by substituting $x = 1$ and $y = 0$ into the equation for $\frac{dy}{dx}$ from part (a):

$$m = \frac{5 \cdot 0 - 2 \cdot 1 \cdot 0^3 - 1}{3 \cdot 1^2 \cdot 0^2 - 5 \cdot 1} = \frac{1}{5}.$$

Then the tangent line is

$$y = \frac{1}{5}(x - 1) = \frac{1}{5}x - \frac{1}{5}$$

2) Differentiate the following, but don't simplify:

(a) (5) $f(x) = \ln(\cos(x) + \sin(3x))$

Solution: By the rule $\frac{d}{dx}\ln(u) = \frac{1}{u} \cdot \frac{du}{dx}$, which incorporates the chain rule (and then the chain rule again on the $\sin(3x)$):

$$f'(x) = \frac{1}{\cos(x) + \sin(3x)} \cdot (-\sin(x) + \cos(3x) \cdot 3)$$

(b) (5) $g(x) = \tan^{-1}(e^{5x}) + \sin^{-1}(x^2)$

Solution: By the derivative rules for the inverse tangent

$$\frac{d}{dx}\tan^{-1}(u) = \frac{1}{1+u^2}\cdot\frac{du}{dx}$$

with $u = e^{5x}$ and the inverse sine:

$$\frac{d}{dx}\sin^{-1}(u) = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$$

with $u = x^2$ (note that these incorporate the chain rule):

$$g'(x) = \frac{5e^{5x}}{1+e^{10x}} + \frac{2x}{\sqrt{1-x^4}}.$$

General Comment: If you lost a lot of points on this one, carefully review the formulas used here. See the class handout sheet from November 13 for the clearest statements. These are also included (along with a lot of other formulas you will not be responsible for) in Sections 3.8 and 3.9 of the book.

3) (10) Water is being poured into a circular cylinder tank with constant radius $r = 5$ meters. If the height of the water in the tank is increasing at a rate of 1 meter per minute, what is the rate of change of the volume of the water in the tank? (The volume of a circular cylinder of radius r and height h is $V = \pi r^2 h$.)

Solution: The volume of water inside the tank is also a cylinder with constant radius $r = 5$ and height increasing as the water is poured in. We have $V = \pi r^2 h = 25\pi h$, so taking derivatives with respect to time:

$$\frac{dV}{dt} = 25\pi\frac{dh}{dt}.$$

Since the height of the water in the tank is increasing at 1 meter per minute, $\frac{dh}{dt} = 1$, and hence

$$\frac{dV}{dt} = 25\pi \cdot 1 = 25\pi \text{ cubic meters per minute.}$$