Linear functions

MATH 133 – Calculus with Fundamentals, section 1, Prof. Little

June 10, 2015

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- The graph $y = mx + b$ is a (non-vertical) *straight line*, and this is the *slope-intercept form* of the equation of a line.
- The constant $b = f(0)$, so the line contains y-axis *intercept* point $(0, b)$.
- The constant m is called the *slope* ("rise over run").
- Meaning of m: Let $(x_1, y_1) = (x_1, mx_1 + b)$ and $(x_2, y_2) = (x_2, mx_2 + b)$ be *any two distinct points* on the graph $y = mx + b$ (so $x_1 \neq x_2$).

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$$\begin{aligned}\frac{y_2 - y_1}{x_2 - x_1} &= \frac{(mx_1 + b) - (mx_2 + b)}{x_2 - x_1} \\ &= \frac{mx_1 - mx_2}{x_2 - x_1} \\ &= \frac{m(x_2 - x_1)}{x_2 - x_1} \\ &= m.\end{aligned}$$

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- This gives the *key property of linear functions*: If we change x to $x + \Delta x$, Δy is always *the same multiple* $\Delta y = m\Delta x$ (it doesn't depend on what x is).

Geometric meaning

This is really just another way of saying that a straight line in the plane is “straight” – this diagram from p. 12 of our text “says it all” – no matter which points (x_1, y_1) and (x_2, y_2) on the line we take, the value of m is the same:

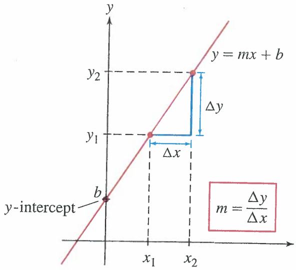

FIGURE 1 The slope m is the ratio “rise over run.”

Example – Telling whether a function is linear

In a previous screencast, we looked at table of values for a function giving the measured period T of a pendulum as a function of the length L:

$L(cm)$	20	30	40	50
$T(sec)$	0.9	1.1	1.27	1.42

Question: Is T *linear*, or perhaps *approximately linear* as a function of L?

Example, continued

Here L values correspond to the x's in the equation for a line (the L is the "independent variable") and the T values are the y's (T is a function of L). Say the four entries from the table are $(L_1, T_1), \ldots, (L_4, T_4)$. With successive pairs:

$$\begin{aligned}
\frac{T_2 - T_1}{L_2 - L_1} &= \frac{1.1 - 0.9}{30 - 20} = .02 \\
\frac{T_3 - T_2}{L_3 - L_2} &= \frac{1.27 - 1.1}{40 - 30} = .017 \\
\frac{T_4 - T_3}{L_4 - L_3} &= \frac{1.42 - 1.27}{50 - 40} = .015.
\end{aligned}$$

First Observation: Since the $\frac{\Delta T}{\Delta L}$ are not constant, the four data points (L_i, T_i) *do not lie on any one line* – the function is *not (exactly) linear*.

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- But in fact the slope values are all *small* and the differences are *very small* – indeed, the $\frac{\Delta T}{\Delta L}$ values all round to one decimal place as $m \doteq .2$ (approximately .2). So the data certainly *is approximately linear*:

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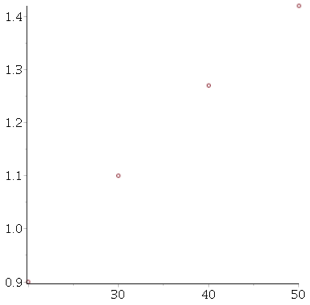
- Here is a point plot of the (L_i, T_i). It's actually quite hard to tell visually that these are *not* on one line:



Comments, continued

- Nevertheless, $\frac{\Delta T}{\Delta L}$ seems to be steadily decreasing as L increases, so this is a good hint that T is *not a linear function of* L. (If the data was essentially linear, but contained experimental error or other "noise" that made the points noncollinear, the $\frac{\Delta T}{\Delta L}$ values would vary randomly on both sides of the actual slope.)

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- *We should not draw any conclusions* without more data points or a theoretical analysis of the physics involved.
- In a physics course, you see that T proportional to $\sqrt{L}$:

$$T = 2\pi\sqrt{\frac{L}{g}},$$

where g is the constant acceleration of gravity on the surface of the earth (i.e. 9.8 m/sec^2 or about 16 ft/sec^2). So this function *is actually not linear*.