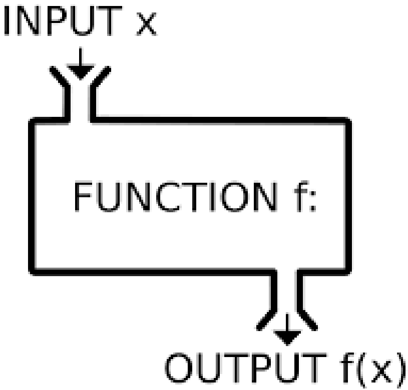
Functions, Tables and Graphs

MATH 133 – Calculus with Fundamentals, section 1, Prof. Little

June 10, 2015

Functions – a conceptual view



- The "name" of the function is f
- Inputs x come from some *domain* (for us – usually a set of real numbers)
- Outputs in some other specified set (for us – also usually a set of real numbers)
- **Key point**: For each x in the domain, there is *exactly one output*, denoted by $f(x)$.

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- **Example**: The period T in seconds of a pendulum (with a weight of a fixed mass) is a function of the length L in centimeters of the pedulum – $L(T)$ is the period in seconds if the length is L centimeters.

Functions described by formulas

You have no doubt seen this sort of function many times: Let f be the function defined by

$$f(x) = \frac{x^2 - 1}{x^2 + 3x}.$$

What this means: We find $f(x)$ by "plugging a value for x into the formula"

$$\begin{aligned} x = -1 \Rightarrow f(-1) &= \frac{(-1)^2 - 1}{(-1)^2 + 3 \cdot (-1)} = \frac{0}{-2} = 0 \\ x = 5 \Rightarrow f(5) &= \frac{(5)^2 - 1}{(5)^2 + 3 \cdot 5} = \frac{24}{40} = \frac{3}{5} = 0.6 \\ x = a^3 \Rightarrow f(a^3) &= \frac{(a^3)^2 - 1}{(a^3)^2 + 3 \cdot a^3} = \frac{a^6 - 1}{a^6 + 3a^3}. \end{aligned}$$

Example, continued

Still considering the function

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- You may have noticed that there are some values where "plugging in" leads to a problem, since $x^2 + 3x = x(x + 3) = 0$ when $x = 0$ or $x = -3$.

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- For this reason, we must *leave* $x = 0, -3$ *out of the domain* of this function, but any other real $x \neq 0, -3$ is OK.

“Rule of thumb” on domains

Unless a particular domain is specified for a function defined by a formula, the domain is the set of *all real x for which the formula makes sense*.

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- But also, $x \neq 3$ or else the denominator is zero.
- All other x are OK, so the domain is the set of all x with $-1 < x < 3$, or $x > 3$ (or the union of the two open intervals $(-1, 3)$ and $(3, +\infty)$).

Tables of values

Especially in experimental sciences, we might need to deal with an (incomplete) description of a function by a *table of values*. Here we have the measured period T of a pendulum as a function of the length L:

$L(cm)$	20	30	40	50
$T(sec)$	0.9	1.1	1.27	1.42

Typical questions: Does T look *linear*, or *approximately linear* as a function of L? If so, what line *fits the data best*?

The graph of a function

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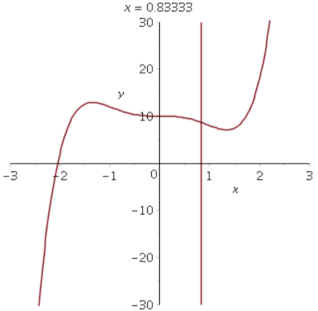
- In other words, the points that are on the graph are those of the form $(x, f(x))$ – the y-coordinate *equals* the function value at the x-coordinate.
- If f is the function with $f(x) = x^2 + 1$, for instance $(-3, 10)$ is on the graph since $f(-3) = (-3)^2 + 1 = 10$
- But $(1, 4)$ is *not on the graph* of $f(x) = x^2 + 1$, since $f(1) = (1)^2 + 1 = 2 \neq 4$.

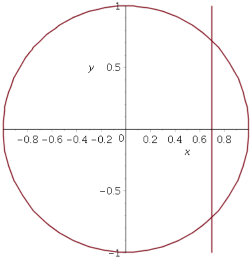
Graphs



- This is the graph of $f(x) = x^5 - 3x^3 + 10$, for x with $-3 \leq x \leq 3$
- For each $x = a$ in the domain, there is *exactly one output* $f(x)$, so *vertical lines* $x = a$ *meet the graph* just once.
- Often say – “the graph passes the *vertical line test*”

Other graphs

Other graphs such the *unit circle* with center at $(0, 0)$, the set of (x, y) with $x^2 + y^2 - 1 = 0$, do not "pass the vertical line test" and are not graphs of functions:



But the upper and lower semicircles, *separately*, are graphs(!)

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- The equation $y = +\sqrt{1 - x^2}$ gives the top half of the circle, and $y = -\sqrt{1 - x^2}$ gives the bottom half.
- The domain of $f_\pm(x) = \pm\sqrt{1 - x^2}$ is the set of x with $1 - x^2 \geq 0$, so $-1 \leq x \leq 1$.