Mathematics 173 – Applied Mathematics 1
Review Sheet for Final Exam
November 30, 1999

General Information

The final exam for this course will be given at 8:30 am on Friday, December 17 (not quite the last possible time!) The exam will be comprehensive, covering all the material we have discussed this semester. I will write the exam so that you should be able to complete it in 2 hours if you work steadily. However, you will have the full 3 hour period if you need more time. Because there are a relatively large number of different PDE, integral formulas, etc. you will need to apply, this will be an *open book, open class notes* exam. I will also supply a table of integrals for you to use. Be sure to prepare carefully, though, because *you will not have enough time* if you need to start every problem by looking up the topic in the book and/or notes.

Topics to Be Covered

These are:

1) Vector Calculus – line integrals, Green's Theorem, and the "alternate form", surface integrals, the Divergence Theorem. Know the precise statements of the two theorems and how to compute both sides in the equalities. (You should know the standard parametrizations for curves like lines, graphs ($y = f(x), x = g(y)$), circles, and ellipses, and surfaces like planes and spheres. Also be prepared to use polar, cylindrical, or spherical coordinates to evaluate some of the double and triple integrals that come up.)
2) PDE in general. Know how to verify that given functions are solutions.
3) The 1D, 2D, and 3D heat equations. Equilibrium solutions for heat equations (know how to derive them, and be able to interpret what they mean). Laplace's Equation for equilibrium solutions of the 2D heat equation. Know the forms of the Laplacian in rectangular and polar coordinates and how to apply them.
4) Boundary value problems for the 1D Heat Equation – know how to express boundary conditions such as "the ends of the rod are maintained at zero temperature", "the ends of the rod are insulated", "the heat flux at the ends is ... " as equations.
5) Separation of variables for general homogenous linear PDE, solutions of the ordinary differential equation $\frac{d^2\phi}{dx^2} = -\lambda\phi$, superposition of solutions.
6) Fourier Series, the Pointwise Convergence Theorem and its consequences, Fourier Sine and Cosine series on $[0, L]$, and their relation with odd and even periodic extensions.
7) The 1D and 2D Wave Equation, boundary value problems, normal modes of vibration for strings and rectangular membranes with zero boundary conditions on the vertical displacement function. Fourier series solutions.

Review Session

I would be happy to hold an evening review session before the exam.

Some Review Problems

I. Let $\mathbf{F} = (xy - y, xy^2)$. Let D be the region in the plane bounded by the line $y = -x$ and the parabola $y = 1 - x^2$, and let α be the positively oriented boundary of D. Show that the conclusion of Green's Theorem is valid by computing both sides of the equation.

II. Show using the "alternate form" of Green's Theorem that if f, g are any two functions with continuous second order partial derivatives on the region D bounded by a simple closed curve α, then

$$\int_\alpha f\nabla g \cdot \mathbf{N}\ ds = \iint_D f\mathrm{div}(\nabla g) + \nabla f \cdot \nabla g\ dA$$

(Here $\mathbf{N}$ is the outward unit normal to α.)

III. Let $\mathbf{F} = (y^2, x + 4y + z, x^2 - z)$, let Φ be the surface formed by the lower hemisphere of the sphere of radius 1 centered at (0,0,0), and the disk in the xy-plane inside $x^2 + y^2 = 1$. Say why the Divergence Theorem applies in this case, and compute both sides in the equality, showing they are the same.

Consult the review sheet for the first midterm and the calculus textbooks on reserve in the Science Library for additional problems of these kinds.

From the text:

1.4.1, 1.4.6, 1.4.7, 1.5.9 (Assume constant thermal properties, assume the temperature is radially symmetric, and that there are no heat sources. Use polar coordinates!)
1.5.12, 1.5.13,
2.3.1-8,
2.4.1,
2.5.1,
3.2.1-4,
3.3.1-3,3.3.18,
4.4.1,4.4.7,4.4.8