Mathematics 174 – Applied Mathematics 2
A Short Table of Fourier Transforms
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Notes

This table is constructed using the following convention for the Fourier transform:

$$\mathcal{F}[f](\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(x) e^{i\omega x}\, dx$$

and the Fourier inversion formula in the form

$$f(x) = \int_{-\infty}^{\infty} \mathcal{F}[f](\omega) e^{-i\omega x}\, d\omega.$$

You will find similar tables in many places, but *beware in comparing what's here with those other tables.* There are MANY different ways people choose to incorporate factors of 2π in the definition of $\mathcal{F}$ and $\mathcal{F}^{-1}$, and most people write the exponential in $\mathcal{F}$ with a negative sign rather than a positive. Depending how all that is done, the results will change accordingly. The first column of the table gives a time domain function or combination of functions. The second column gives the corresponding Fourier transform. If you are clever, you can also see how to compute direct Fourier transforms by reading this table from right to left(!) (Hint: what is $\mathcal{F}(\mathcal{F}[f])$ if f is suitably nice?)

Table – Special Functions

$f(x)$	$\xrightarrow{\mathcal{F}}$	$\mathcal{F}[f](\omega)$
$\Pi_a(x) = \begin{cases} 1 & \text{if } \lvert x\rvert \le a \\ 0 & \text{otherwise} \end{cases}$	$\to$	$a\text{sinc}(a\omega) = \frac{\sin(a\omega)}{\pi\omega}$
$\Lambda_a(x) = \begin{cases} 2a - \lvert x\rvert & \text{if } \lvert x\rvert \le 2a \\ 0 & \text{otherwise} \end{cases}$	$\to$	$2\pi a^2 \text{sinc}^2(a\omega) = \frac{2\sin^2(a\omega)}{\pi\omega^2}$
e^{-x^2}	$\to$	$\frac{1}{\sqrt{4\pi}} e^{-\omega^2/4}$
$\delta(x)$	$\to$	$\frac{1}{2\pi}$
$e^{-\lvert x\rvert}$	$\to$	$\frac{1}{\pi}\frac{1}{1+\omega^2}$

(continued on back)

Table – General Properties

$(Linearity)$	$cf(x) + g(x)$	$\rightarrow$	$c\mathcal{F}[f](\omega) + \mathcal{F}[g](\omega)$
$(Scaling)$	$f(ax)$	$\rightarrow$	$\frac{1}{\lvert a \rvert}\mathcal{F}[f](\omega/a)$
$(Shifting)$	$f(x+a)$	$\rightarrow$	$\mathcal{F}[f](\omega)e^{-ia\omega}$
$(Modulation)$	$f(x)e^{iax}$	$\rightarrow$	$\mathcal{F}[f](\omega + a)$
$(Convolution)$	$(f * g)(x)$	$\rightarrow$	$2\pi\mathcal{F}[f](\omega)\mathcal{F}[g](\omega)$
$(Derivatives)$	$f^{(n)}(x)$	$\rightarrow$	$(-i\omega)^n\mathcal{F}[f](\omega)$