Mathematics 174 – Applied Mathematics 2
Problem Set 5
March 30, 2000

I. Compute the Fourier Transforms of each of the following functions using the indicated methods)

A) (by direct computation)

$$f(x) = \begin{cases} 1 & \text{if } |x| \leq 1 \\ 2 & \text{if } 1 < |x| < 2 \\ 0 & \text{otherwise} \end{cases}$$

B) (by direct computation)

$$f(x) = \begin{cases} \sin(x) & \text{if } |x| \leq \frac{\pi}{2} \\ 0 & \text{otherwise} \end{cases}$$

C) Show that if $xf(x)$ is in $L^1(\mathbf{R})$, then

$$\mathcal{F}[xf(x)](\omega) = -i\frac{d}{d\omega}\mathcal{F}[f](\omega)$$

and use that to evaluate the Fourier Transform of $f(x) = xe^{-x^2}$.

D) (by using the general scaling and modulation properties)

$$f(x) = \cos(4x)e^{-3x^2}$$

(*Answer:* $\frac{\sqrt{3}}{12\sqrt{\pi}}\left(e^{-(\omega+4)^2/12} + e^{-(\omega-4)^2/12}\right)$)

E) (by using the Fourier Inversion theorem and a known transform) $f(x) = \frac{1}{1+x^2}$

II. For real numbers a and c, let

$$g_{a,c} = \frac{1}{\sqrt{2\pi a^2}}e^{-(x-c)^2/(2a^2)}$$

(a scaled shifted Gaussian).

A) What are the meanings of the constants a, c in terms of the "bell-shaped" curve $y = g_{a,c}(x)$?

B) Show that if a, c, b, d are any real numbers, then

$$g_{a,c}(x) * g_{b,d}(x) = g_{\delta,c+d}(x)$$

where $*$ = convolution, and $\delta = \sqrt{a^2 + b^2}$. (Hint: Take the Fourier transform of the convolution, work with what you get, then take the inverse transform.)

III. Let $f : \mathbf{R} \to \mathbf{C}$ be an L^1 function, and let F denote its Fourier transform. Show that if $F(\omega) = 0$ for all $|\omega| > \omega_0$, then

$$f(x) * \frac{\sin(ax)}{\pi x} = f(x)$$

for all $a > \omega_0$.

Due: In class, Thursday, April 6.