

College of the Holy Cross, Fall Semester, 2018  
MATH 351, Midterm 2  
Friday, November 16

Your Name: \_\_\_\_\_

**Instructions:** Please show all work necessary to justify your answers. Use the back of the preceding page if you need more space for scratch work. There are 100 possible points distributed as below.

Please do not write in the space below

Problem	Points/Poss
I	/ 25
II	/ 30
III	/ 25
IV	/ 20
Total	/100

I. Let  $G = U(20)$  (where the operation is multiplication mod 20), and  $N = \langle 9 \rangle$  in  $G$ .

(A) (5) How do you know is  $N$  a normal subgroup of  $G$ ?

(B) (20) Construct a group table for the factor (quotient) group  $G/N$ . To which “standard” group is this isomorphic?

II. (A) (10) Let  $\alpha : G \rightarrow H$  be a group homomorphism. Show that  $\ker(\alpha)$  is a normal subgroup of  $G$ .

(B) (10) State the First Isomorphism Theorem for groups.

(C) (10) Let  $G = \mathbb{Z} \times \mathbb{Z}$  and  $N = \{(a, 2a) \mid a \in \mathbb{Z}\}$ . Using the First Isomorphism Theorem, determine a group isomorphic to  $G/N$ .

III. Let  $G$  be a group of order 14.

- (A) (15) Show that  $G$  contains elements of order 2 and elements of order 7. You may use without proof any general facts we know that apply here.

(B) (10) Still assuming  $G$  has order 14, any element  $a$  of order 7 generates a normal subgroup. If  $b$  has order 2, determine all possibilities for  $bab = bab^{-1}$ .

IV. (A) (10) Using the Fundamental Theorem, give a complete list of abelian groups of order 72 up to isomorphism.

(B) (5) Let  $G = \mathbb{Z}_4 \times \mathbb{Z}_{18}$ . To which group in your list from part (A) is  $G$  isomorphic?

(C) (5) Let  $G = \langle a \rangle$  be a cyclic group of order 72. Write  $a = b \cdot c$ , where  $b$  is a 2-element of  $G$  and  $c$  is a 3-element of  $G$ .