College of the Holy Cross, Fall Semester, 2018
MATH 351, Midterm 1
Thursday, October 4

Your Name: ______________________________

Instructions: Please show all work necessary to justify your answers. Use the back of the preceding page if you need more space for scratch work. There are 100 possible points distributed as below.

Please do not write in the space below

Problem	Points/Poss
I	/ 25
II	/ 30
III	/ 25
IV	/ 20
Total	/100

I. Let $G = \mathrm{SL}(2, \mathbb{Z})$, the set of 2×2 integer matrices with determinant 1, which is a group under the operation of matrix multiplication. Let

$$H = \left\{ A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in G \mid b \equiv 0 \bmod 4 \right\}.$$

(A) (15) Is H a subgroup of G? Why or why not?

(B) (10) Compute this matrix product, noting that the left and right matrices are inverses of each other and the middle matrix is in H:

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 8 \\ 1 & 9 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}$$

What does the result tell you about the subgroup H?

II. (A) (15) Let $G = \langle a \rangle$ be a cyclic group. Prove that every subgroup of G is also cyclic.

(B) (5) In part (A), suppose that a has order 120. List all the integers that are orders of elements of G.

(C) (10) Still assuming a has order 120, how many elements of G have order 24? What are they?

III. (A) (10) Let H be a subgroup of a group G. Show that $aH = bH$ if and only if $a^{-1}b \in H$.

(B) (15) Let $G = S_3$ and let

$$H = \left\langle \begin{pmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{pmatrix} \right\rangle .$$

Find all of the left and right cosets of H in G.

IV. Let G be a group containing subgroups H and K with $|H| = 28$ and $|K| = 65$.

(A) (10) What is the smallest possible value for $|G|$?

(B) (10) Show that $H \cap K = \{e\}$.