College of the Holy Cross, Fall 2016
MONT 106Q – Mathematical Thinking
Final Exam, December 13, 2016

Your Name: _____

Directions

Do all work on the sheets provided (if you use the back of a sheet, please place a note telling me to look there). There is an extra blank sheet of paper at the end that you can use either as scratch paper or as extra space for your essay. You may detach that if you like, but please put your name on it and hand it in with your exam if you do detach it. The numbers in parentheses next to the questions are there point values (100 points total).

Please do not write in the space below

Problem	Points/Poss
I	/ 10
II	/ 10
III	/ 10
IV	/ 10
V	/ 10
VI	/ 10
Essay	/ 40
Total	/100

I. A) (5) Express the base 10 number 457 in base 2.

B) (5) What is the next integer after 111111 in base 2?

II. Compute "the Egyptian way"

A) (5) 53×123

B) (5) $301 \div 21$ (that is, "calculate with 21 to yield 301"; for the fractional part, get a sum of distinct Egyptian fractions any way you can, using the $2/n$ table provided on the accompanying sheet.)
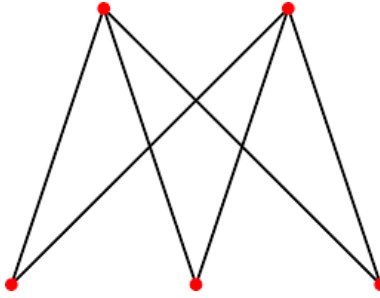
Figure 1: The graph $K_{2,3}$

III. Let m, n be two integers ≥ 1. The graph $K_{m,n}$ has two groups of vertices, one containing m vertices and one containing n vertices. There are edges containing each of the vertices in the first group to each of the vertices in the second, and no other edges. For instance one of these graphs is shown in the figure above.

A. (5) Does the graph in the figure have Eulerian circuits? Why or why not?

B. (5) For which pairs m, n does $K_{m,n}$ have an Eulerian circuit? Explain your reasoning.

IV. (10) Suppose you are playing *mancala* and it is your turn. Your pits contain numbers of stones left to right as follows: $6, 4, 2, 2, 0, 0$. Is it possible to get all 14 stones into your store before the other player gets to play? If so, how would you do it, step by step? If not, why not?

Figure 2: Figure for question V, part C

V. Multiple Choice. Circle the correct arabic-number option for each question.

A. (2) In this course we saw examples of the mathematical structure of a *group* in

    I. analyzing the Warlpiri kinship system

    II. considering the game of *mancala*

    III. studying symmetries of strip patterns

    (1) I only

    (3) III only

    (4) I and III only

    (5) I,II,and III

B. (2) Which Mayan number symbols represent the base-10 number 383?

    (1)

    (2)

    (3)

    (4)

    (5)

C. (2) What is the type of the symmetric strip pattern in Figure 2?

    (1) $p111$

    (2) $pm11$

    (3) $pma2$

    (4) $pmm2$

D. (2) Which of the following combinations of the mother m and father f relations is the same as fm in the Warlpiri kinship system?

    (1) mf

(2) $m^2 f$

(3) $m^3 f$

(4) mfm

E. (2) In *Flatland*, who was the originator of the phrase "configuration makes the man."

(1) the Sphere

(2) the Square

(3) Chromatistes

(4) Pantocyclus

VI. Short answer. Answer *any five* of the following. If you answer more than five, only the best five will be used.

A) (2) Draw a diagram representing which sections are allowed to marry in the Warlpiri kinship system and how the section of the children is determined.

B) (2) Why is the Maya number system called a "mixed-base" system? Explain.

C) (2) Apart from as entertainment, how was the game of *mancala* used in some traditional African societies?

E) (2) What is the main difference between $pmm2$ and $pma2$ symmetric strip patterns?

F) (2) From what part of the world do the textiles we saw in the Cantor Art Gallery come?

G) (2) Is it possible for a symmetric strip pattern to have reflection symmetry across the main translation axis (usually horizontal) and no glide reflection symmetries? Explain.

IV. Essay. (40) Pick one of the two following topics. Say clearly which one you are addressing.

Option 1: What are some of the borders/boundaries that Shakespeare uses to structure his plot and characters in *Othello*? For instance, what is a "Moor?" What does the word mean, and how does the fact that Othello is a Moor create a boundary between him and Venetian society? Where does the action of the play take place, and how does that include a political or physical border? Who besides Othello has transcended borders and how? Does transcending those borders end well or result in tragedy?

Option 2: A key episode in *Flatland* invlives the interaction between the Sphere from Spaceland and the Square from Flatland where the border of the two-dimensional Flatland is literally transcended. What does the Sphere do to demonstrate the existence of a third dimension? Why does the Sphere say, "Listen, no stranger must witness what you have witnessed"? What is the Square's reaction to being taken into space? Find three words in the text that have the prefix *omni-* and explain what they mean and how they relate to this episode. Why does the Sphere say that omnividence does not make a person a God? What qualities does he say make one more divine? What is the Square's reaction?