College of the Holy Cross, Fall 2016
MONT 106Q – Mathematical Thinking
Solutions for Final Exam Questions, December 13, 2016

I. A) (5) Express the base 10 number 457 in base 2.

Solution: The number $457 = 256 + 128 + 64 + 8 + 1$, so the base 2 form is 111001001.

B) (5) What is the next integer after 111111 in base 2?

Solution: It's 1000000. (You can see this either by adding 1 as described in the solutions for the practice problems or converting to base 10: $111111 = 63$, so the next number is 64, which is 1000000.)

II. Compute "the Egyptian way"

A) (5) 53×123

Solution: We double repeatedly

$$\begin{aligned} \Rightarrow 1 \times 123 &= 123 \\ 2 \times 123 &= 246 \\ \Rightarrow 4 \times 123 &= 492 \\ 8 \times 123 &= 984 \\ \Rightarrow 16 \times 123 &= 1968 \\ \Rightarrow 32 \times 123 &= 3936 \end{aligned}$$

Since $53 = 32 + 16 + 4 + 1$, the product is $3936 + 1968 + 492 + 123 = 6519$.

B) (5) $301 \div 21$ (that is, "calculate with 21 to yield 301"; for the fractional part, get a sum of distinct Egyptian fractions any way you can, using the $2/n$ table provided on the accompanying sheet.)

Solution: We double 21 repeatedly until the next doubling takes us past 301:

$$\begin{aligned} 1 \times 21 &= 21 \\ \Rightarrow 2 \times 21 &= 42 \\ \Rightarrow 4 \times 21 &= 84 \\ \Rightarrow 8 \times 21 &= 168. \end{aligned}$$

So $301 = 14 \times 21 + 7$. The remaining part of the computation is to write $\frac{7}{21}$ in terms of Egyptian fractions. But that is easy: $301 \div 21 = 14 + \frac{1}{3}$.

III. Let m, n be two integers ≥ 1. The graph $K_{m,n}$ has two groups of vertices, one containing m vertices and one containing n vertices. There are edges containing each of the vertices in the first group to each of the vertices in the second, and no other edges. For instance one of these graphs is shown in the figure above.

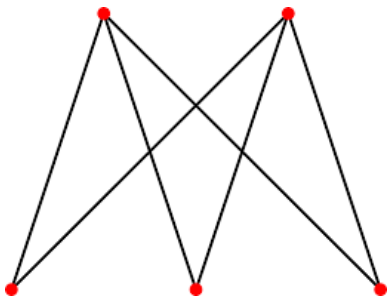

Figure 1: The graph $K_{2,3}$

A. (5) Does the graph in the figure have Eulerian circuits? Why or why not?

Solution: There is an Eulerian path starting at one of the top vertices and ending at the other, but that is not an Eulerian circuit. There are not any of those because not all of the vertices have even degree.

B. (5) For which pairs m, n does $K_{m,n}$ have an Eulerian circuit? Explain your reasoning.

Solution: An Eulerian circuit exists if and only if m, n are *both even*. The reason is that each of the m vertices in the first group has degree n and each of the n vertices in the second group has degree m. Both of those must be even for the $K_{m,n}$ graph to have an Eulerian circuit.

IV. (10) Suppose you are playing *mancala* and it is your turn. Your pits contain numbers of stones left to right as follows: $6, 4, 2, 2, 0, 0$. Is it possible to get all 14 stones into your store before the other player gets to play? If so, how would you do it, step by step? If not, why not?

Solution: Playing from the rightmost harvestable pit (underlined) each time we would proceed as follows:

$$\underline{6}, 4, 2, 2, 0, 0$$
$$0, 5, 3, 3, 1, \underline{1}$$
$$0, \underline{5}, 3, 3, 1, 0$$
$$0, 0, 4, 4, 2, \underline{1}$$
$$0, 0, 4, 4, \underline{2}, 0$$
$$0, 0, 4, 4, 0, \underline{1}$$
$$0, 0, \underline{4}, 4, 0, 0$$
$$0, 0, 0, 5, 1, \underline{1}$$
$$0, 0, 0, 5, 1, 0$$

and we are stuck. This is not one of the "good positions." $6, 4, 2, 0, 0, 0$ is the "closest" one that does work, in a way.

Figure 2: Figure for question V, part C

V. Multiple Choice. Circle the correct arabic-number option for each question.

A. (2) In this course we saw examples of the mathematical structure of a *group* in

I. analyzing the Warlpiri kinship system

II. considering the game of *mancala*

III. studying symmetries of strip patterns

(3) I and III only

B. (2) Which Mayan number symbols represent the base-10 number 383?

(1)

C. (2) What is the type of the symmetric strip pattern in Figure 2?

(2) $pm11$ (because it has no rotation symmetries, but it does have vertical reflection symmetry)

D. (2) Which of the following combinations of the mother m and father f relations is the same as fm in the Warlpiri kinship system?

(3) m^3f

E. (2) In *Flatland*, who was the originator of the phrase "configuration makes the man."

(4) Pantocyclus

VI. Short answer. Answer *any five* of the following. If you answer more than five, only the best five will be used.

A) (2) Draw a diagram representing which sections are allowed to marry in the Warlpiri kinship system and how the section of the children is determined.

Solution: See the diagram on page 71 of *Ethnomathematics* or your class notes.

B) (2) Why is the Maya number system called a "mixed-base" system? Explain.

Solution: It is a mixed-base system because while it is mostly base-20, the third digits of numbers represent numbers of $18 \times 20 = 360$'s rather than numbers of $20^2 = 400$'s. The following digits continue from there: $18 \times 20^2, 18 \times 20^3$, etc.

C) (2) Apart from as entertainment, how was the game of *mancala* used in some traditional African societies?

Solution: It was reportedly used as a test of strategic thinking for prospective chiefs.

E) (2) What is the main difference between $pmm2$ and $pma2$ symmetric strip patterns?

Solution: The main difference is that $pmm2$ patterns have horizontal reflection symmetry, while $pma2$ patterns do not.

F) (2) From what part of the world do the textiles we saw in the Cantor Art Gallery come?

Solution: From the island of Sarawak in Indonesia (south Asia).

G) (2) Is it possible for a symmetric strip pattern to have reflection symmetry across the main translation axis (usually horizontal) and no glide reflection symmetries? Explain.

Solution: No, this is not possible. The transformation obtained by translating by one unit along the main axis, followed by reflection across that axis must also be in the symmetry group, and that is a glide reflection.

VII. Essay. (40) Pick one of the two following topics. Say clearly which one you are addressing.

Option 1: What are some of the borders/boundaries that Shakespeare uses to structure his plot and characters in *Othello*? For instance, what is a "Moor?" What does the word mean, and how does the fact that Othello is a Moor create a boundary between him and Venetian society? Where does the action of the play take place, and how does that include a political or physical border? Who besides Othello has transcended borders and how? Does transcending those borders end well or result in tragedy?

Option 2: A key episode in *Flatland* involves the interaction between the Sphere from Spaceland and the Square from Flatland where the border of the two-dimensional Flatland is literally transcended. What does the Sphere do to demonstrate the existence of a third dimension? Why does the Sphere say, "Listen, no stranger must witness what you have witnessed"? What is the Square's reaction to being taken into space? Find three words in the text that have the prefix *omni-* and explain what they mean and how they relate to this episode. Why does the Sphere say that omnividence does not make a person a God? What qualities does he say make one more divine? What is the Square's reaction?