College of the Holy Cross, Fall 2019
MATH 110-02 – Algebra Through History
Final Exam, December 18, 2019

Your Name: _____

Directions

Do all work on the sheets provided (if you use the back of a sheet, please place a note telling me to look there). There is an extra blank sheet of paper at the end that you can use either as scratch paper or as extra space for your essay. You may detach that if you like, but please put your name on it and hand it in with your exam if you do detach it. The numbers in parentheses next to the questions are there point values (100 points total).

Please do not write in the space below

Problem	Points/Poss
I	/ 40
II	/ 20
III	/ 40
Total	/100

Have a peaceful and joyous holiday season!

I. Answer any 8 of the following 12 short answer questions. If you submit answers for more than 8, only the best 8 will be used to compute your score.

A) (5) Give a *brief* description of the number system used by the Old Babylonian scribes (one sentence would suffice). Explain and/or give an example of one ambiguous feature of this system.

B) (5) According to Jens Høyrup, what is the best way to describe the operations in the solution of the problem from the Old Babylonian tablet YBC 6967? (You don't need to reproduce the whole solution; just describe in general terms.)

C) (5) Approximately when and where do we think Euclid was active? What evidence is there for this or any details of his life?

D) (5) What effect did the discovery of *incommensurable magnitudes* apparently have on the presentation of basic mathematics in Euclid's *Elements*? Explain briefly.

E) (5) One translation of Proposition 2 from Book II of Euclid's *Elements* is as follows: "If a straight line is cut at random, then the sum of the rectangles contained by the whole and each of the segments equals the square on the whole." What algebraic equation is equivalent to this if a straight line of length x is cut into segments of lengths y, z, w?

F) (5) Book II of Euclid's *Elements* has often been described as "geometric algebra." What historian that we discussed disputes this and why? Explain *briefly*.

G) (5) What major innovation do we see in Diophantos' *Arithmetica*? Explain briefly.

H) (5) Where does our word *algebra* come from historically? Explain briefly.

I) (5) Approximately when and where was Al-Khwarizmi active?

I) (5) Why did Al-Khwarizmi need to handle "squares equal roots and numbers" quadratic Equations separately from "squares and roots equal numbers" quadratic equations? Explain briefly.

J) (5) What roles did medieval European mathematicians such as Gerbert of Aurillac, Gerard of Cremona, Robert of Chester, and Leonardo of Pisa play in the development of algebra?

K) (5) In what ways were the techniques presented by Viète in his *Introduction to the Analytical Art* an advance over what Diophantos had done with some of the same problems? Explain briefly.

L) (5) The approach to doing geometry introduced by Descartes in *La Géometrie* is often called "analytic geometry" today. What is the historical explanation for this? To what ancient Greek mathematician's work is the word "analytic" primarily referring?

II. Identifications. For any 4 of the following 6 graphics, texts, or formulas, give the name of the period or the mathematician who would be most closely identified with that item, and explain the meaning briefly. If you submit solutions for more than 4, only the best 4 will be used in computing your score on this question.

A) (5)

$$x = \sqrt[3]{\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \sqrt[3]{\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

B) (5) $\Delta^{\Upsilon} 3\varsigma 4 \Lambda M^o 4$

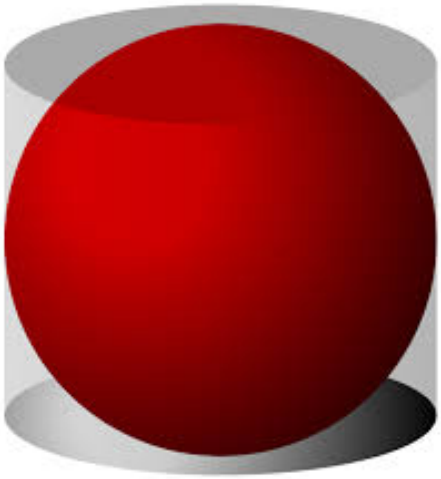

Figure 1: Figure for Question II, part C

C) (5) See Figure 1 above.

D) (5) "The supreme and everlasting law of equations or proportions, which is called the law of homogeneity because it is conceived with respect to homogeneous magnitudes, is this: Only homogeneous magnitudes are to be compared with one another."

E) (5) "I find nothing here so difficult that it cannot be worked out by anyone at all familiar with ordinary geometry *and algebra*, who will consider carefully all that is set out in this treatise."

F) (5) "*Analysis*, then takes that which is sought as if it were admitted and passes from it through its successive consequences to something which is known as the result of synthesis [i.e. things proved before]. For in analysis, we admit that which is sought as if it were already done and we inquire what it is from which this results, and again what is the antecedent cause of the latter, and so on, until by retracing our steps, we come upon something already known or belonging to the class of first principles ... "

III. *Essay* (40) Although this has mainly been a course about the historical development of algebra, when you look back at what we have learned, you should see that one consistent theme throughout has been the ways that algebra and geometry have been connected with one another, and the changing ways that mathematicians have thought about that relationship. Discuss the relationship between algebra and geometry in each of the following periods, or mathematical works.

(1) The Old Babylonian period and problem texts such as YBC 6967. Is there a difference of opinion about how to interpret what is algebra and what is geometry there?

(2) The presentation of elementary mathematics in Euclid's *Elements*, especially in Book II.

(3) The *Arithmetica* of Diophantos.

(4) The *Hisab al-jabr w'al-muqabala* of Al-Khwarizmi.

(5) *La Géometrie* of René Descartes.

In teaching algebra today, which way of dealing with algebra and geometry do you think would help students most to learn the subject? And would knowing some of this history help?