College of the Holy Cross, Fall 2019
MATH 110-02 – Algebra Through History
Midterm Exam, October 25, 2019

Your Name: ________________________________

Directions

Do all work on the sheets provided (if you use the back of a sheet, please place a note telling me to look there). There is an extra blank sheet of paper at the end that you can use either as scratch paper or as extra space for your essay. You may detach that if you like, but please put your name on it and hand it in with your exam if you do detach it. The numbers in parentheses next to the questions are there point values (100 points total).

Please do not write in the space below

Problem	Points/Poss
I	/ 20
II	/ 20
III	/ 20
IV	/ 40
Total	/100

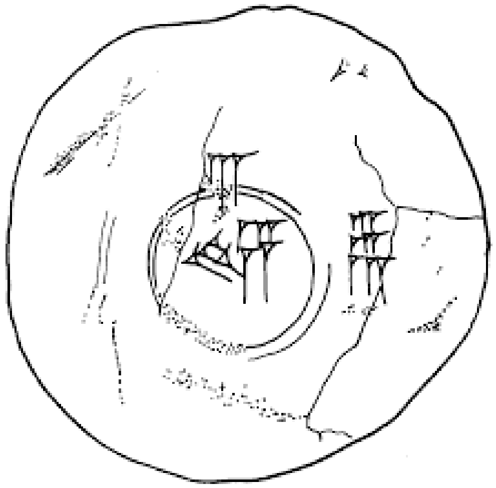

Figure 1: The Old Babylonian tablet YBC 7302

I. A) (15) Refer to Figure 1 above showing an Old Babylonian practice tablet from a scribal school in the city of Susa. It shows three separate numbers in the cuneiform base-60 script. What are those three numbers?

B) (5) From the context (the rough circle with the numbers), it is surmised that this shows a calculation of an approximate area of a circle. Interpret the number written over the circle as the circumference, and the number inside the circle as the area. *Question:* What value is being used for the constant π? Hints: (1) The number inside is not a whole number; you want to think of it as the first digit after the ; in the fractional part. Useful information: the area of a circle of radius r is $A = \pi r^2$ and the circumference is $C = 2\pi r$, so the area is $A = \frac{C^2}{4\pi}$ in terms of the circumference.

II.

A) (5) In Greek mathematics, what did it mean to say that that two magnitudes are *incommensurable*?

B) (15) Give the proof the ancient Greeks found for the fact that the diagonal of a square of side 1 is not commensurable with the side.

III. Short answer. Answer any four of the following. If you answer more than four, you can earn some Extra Credit points.

A) (5) One of Diophantos' propositions contains the following expression $\Delta^{\Upsilon}4M^o16\Lambda\varsigma16$ (containing one unknown). Rewrite this in modern notation, calling the unknown x.

B) (5) What are the approximate dates of the YBC 6967 and YBC 7289 tablets? About when did Euclid live?

C) (5) What is "Fermat's Last Theorem" and what is its connection with Diophantos? When and by whom was this finally solved?

D) (5) In what way is the solution of the problem given on the YBC 6967 tablet related to our modern quadratic formula?

E) (5) Proposition 4 in Book II of Euclid's *Elements* says: *If a straight line is cut at random then the square on the whole line is equal (in area) to the sum of the squares on the pieces together with twice the rectangle contained by the two pieces.* What modern algebraic equation is equivalent to this? Call the lengths of the two pieces x, y.

F) (5) What would be the hardest arithmetic operations to carry out in the Old Babylonian base-60 number system? How did they keep track of all the information they needed for these?

IV. (40) Essay. You have the choice of responding to *either prompt 1 or 2*. State which one you have chosen at the start of your essay.

1) A certain older history of mathematics says, flatly, that "the distinguishing feature of Babylonian mathematics is its algebraic character." Of the historians we have mentioned, who would agree with this claim and who would disagree? Explain using the interpretations your historians would give for the YBC 6967 problem of (what we would phrase as) solving the equation $x = 60/x + 7$.

2) George G. Joseph, the author of another book on the non-European roots of modern mathematics called *The Crest of the Peacock*, offers this overall evaluation of the ultimate impact of Greek geometry: "There is no denying that the Greek approach to mathematics produced remarkable results, but it also *hampered* the subsequent development of the subject. ... Great minds such as Pythagoras, Euclid, and Apollonius spent much of their time creating what were essentially abstract idealized constructs; how they arrived at a conclusion was in some way more important than any practical significance." First, what does the last sentence mean? Would this criticism seem to be apt for Diophantos' *Arithmetica* as well? Is it necessary for all the mathematics we learn and do to have practical usefulness or significance?