Mathematics 36, section 5 – AP Calculus
Discussion 1 – Using Derivatives
September 22, 1999

Working in a Group

Since this is also the first of the discussion classes of the semester, a few words about this way of working are probably in order. In the discussion meetings of this class, we will be aiming for truly *collaborative learning* – that is, for an integrated group effort in analyzing and attacking the discussion questions. The ideal is for everyone in each of the groups to be fully involved in the process. The idea is that, by actively participating in the class through talking about the ideas yourself in your own words, you can come to a better first understanding of what is going on than if you simply listen to someone else (even me!) talk about it.

However, it must be said that to get the most out of this kind of work, you may have to adjust some of your preconceptions. In particular:

- This is *not a competition*. You and your fellow group members are working as a team, and the goal is to have everyone understand what the group does fully.
- At different times, it is inevitable that different people within the group will have a more complete grasp of what you are working on and others will have a less complete grasp. Dealing with this a group setting is excellent preparation for real work in a team; it also offers opportunities for significant educational experiences:
  a) If you feel totally "clueless" at some point and everyone else seems to be "getting it," you need to feel free to ask questions and even pester your fellow group members until the point has been explained to your full satisfaction. (Don't forget, the others may be jumping to unwarranted conclusions, and your questions may save the group from pursuing an erroneous train of thought!)
  b) On the other hand, when you think you do see something, you may need to be willing to explain it carefully to others. (Don't forget, the *absolutely best* way to make sure you really understand something is to try to explain it to someone else. If you are skipping over an important point in your thinking, it can become very apparent when you set out to convey your ideas to a team member.)

  In short, everyone has something to contribute, and everyone will contribute in different ways at different times.

Background

Our techniques for analyzing *critical points* of functions can be applied to many problems, especially optimization problems, where we seek a maximum or minimum value of some function.
(Over for questions)

Discussion Questions

A) The following plot shows the graph $y = f'(x)$ (the *derivative*) for some function f.

Using this information, sketch a qualitative plot first for $f''(x)$. Then use all the information you have to sketch $y = f(x)$, assuming $f(0) = 1$. Say where f is increasing/decreasing, concave up/down, and give the locations of all local max and min and inflection points.

B) To give a patient an antibiotic slowly, the drug is injected into a muscle (rather than directly into a vein). The quantity of the drug in the bloodstream of the patient starts at 0 at $t = 0$ when the injection is given, increases to a maximum, then decays asymptotically to zero. The rate of increase of the quantity of the drug is greatest at the time of the injection.
   1) Sketch a possible graph for the quantity of the drug in the bloodstream as a function of time.
   2) Using that graph, sketch a graph of the *rate of change of the amount of the drug* – the rate at which the drug is entering or leaving the bloodstream.
   3) Construct a third graph showing the rate of change of the rate of change of the quantity of the drug. What is the meaning of the t-axis intercept of this graph?

C) The efficiency of a screw is given by

$$E = \frac{(\theta - \mu\theta^2)}{\mu + \theta}$$

where θ is the pitch angle of the threads of the screw, and μ is a positive constant. What value of θ maximizes E?

D) You have a 10 inch length of flexible metal wire. If you cut the wire into two pieces, bend one to enclose a square, and the other to enclose a circle, what is the *largest* total area you can enclose? Give a full justification of your answer.

Assignment:

One write-up per group of solutions for these problems, due Monday, September 27.