College of the Holy Cross
MATH 134, Calculus With Fundamentals 2
Final Examination – Thursday, May 10

Your Name: _____

Instructions: For full credit, you must show *all work* on the test pages. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values. There are 200 total points on this exam.

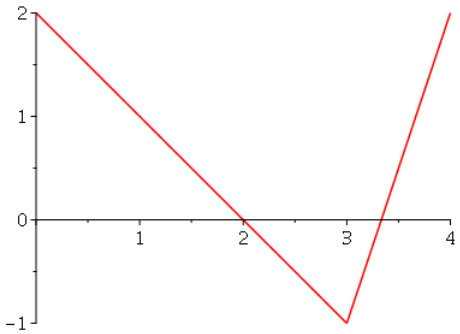
Please do not write in the space below

Problem	Points/Poss
I	/ 20
II	/ 30
III	/ 30
IV	/ 40
V	/ 30
VI	/ 20
VII	/ 30
Total	/ 200

HAVE AN ENJOYABLE AND PRODUCTIVE SUMMER!

I. Integration and the Fundamental Theorem of Calculus.

(A) (10) Let $f(x) = \begin{cases} 2 - x & \text{if } 0 \le x \le 3 \\ 3x - 10 & \text{if } 3 < x \le 4. \end{cases}$ whose graph is shown here:



Let $F(x) = \int_0^x f(t)\,dt$. Complete the following table of values for $F(x)$ and $F'(x)$. (Assume the graph continues to the left and the right so $F'(0)$ and $F'(4)$ make sense.)

x	0	1	2	3	4
$F(x)$					
$F'(x)$					

(B) (10) Compute $G'(x)$ if $G(x) = \displaystyle\int_0^{x^3} e^{-x^2}\,dx$.

$G'(x) =$

II. Compute the following integrals using basic rules, u-substitution, or integration by parts, as appropriate.

(A) (10) $\int x^2 \cos(x^3)\, dx$

Integral = $\boxed{}$

(B) (10) $\int x^2 e^{5x}\, dx$

Integral = $\boxed{}$

(C) (10) $\int x^2(1 - 4x + 3x^2)\, dx$

Integral = $\boxed{}$

III. Trigonometric Substitutions.

A. (10) What trigonometric substitution would you use to evaluate

$$\int \frac{x^2}{\sqrt{49 - x^2}}\, dx?$$

Make the substitution and simplify so there is no square root in the resulting trigonometric integral.

Integral = $\boxed{}$

B. (10) Find $\int \sin^2(\theta)\cos^2(\theta)$ using the trigonometric reduction formulas.

Integral = $\boxed{}$

C. (10) Suppose you had used the trig substitution $x = 3\tan(\theta)$ and you integrated the resulting trigonometric integral to get

$$\theta + \sec(\theta) + C$$

What is the final answer, expressed as a function of x?

Integral = $\boxed{}$

IV. Partial Fractions.

A. (5) How do you recognize the situation where you need to start by using polynomial division? Describe briefly.

B. (10) To decompose $\dfrac{x^2+3}{(x+1)(x+2)^2(x^2+4)}$ into partial fractions, what would the form of the fractions be (leave coefficients as undetermined; *do not try to solve for them*).

C. (10) Determine the values A, B, C making

$$\frac{3x+1}{x(x+3)(x-1)} = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-1}$$

$A =$ $B =$ $C =$

D. (15) You have decomposed a rational function $f(x)$ into partial fractions as

$$f(x) = 3x^3 + 4 + \frac{6}{x} + \frac{2}{x^2} + \frac{2x-3}{x^2+4}.$$

What is $\int f(x)\, dx$?

Integral =

V. All parts of this problem refer to the region R bounded by $y = x$, $y = 4 - x^2$, $x = 0$ and $x = 1$.

A. (10) Sketch the region R.

B. (10) Compute the area of the region R.

Area = $\boxed{}$

C. (10) Compute the volume of the solid of revolution obtained by rotating the region R about the x-axis.

Volume = $\boxed{}$

VI. Other Applications of Integrals.

(A) (10) Find the general solution of the differential equation $\dfrac{dy}{dx} = xe^y$.

General solution = $\boxed{}$

(B) (10) Suppose that a random variable T has pdf $f(t) = 20t^3(1-t)$ for $0 \le t \le 1$ (and 0 otherwise). Find $P\left(0 \le T \le \frac{1}{3}\right)$.

Probability = $\boxed{}$

VII. Miscellany – *answer any 3 of the following 5 questions.* If you answer more than 3, all points received will be counted, giving a possibility of up to 20 points Extra Credit.

(A) (10) The standard normal pdf is the function $f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. Use a left-hand Riemann sum with $N = 5$ to approximate $\int_0^{0.5} f(x)\, dx$.

$L_5 =$ $\boxed{}$

(B) (10) If Z has a standard normal distribution, what is $P(0 \leq Z \leq 0.50)$ from our table? Why is this number smaller than the answer in part A? Explain briefly.

$P(0 \leq Z \leq 0.50) =$ [ ]

(C) (10) What is the future value of a series of monthly payments of \$300 at 5% interest, over a period of 6 years? If you were making these payments to pay off a car loan for \$18000 would you have overpaid or still owe the loan company?

FV of series of payments = [ ]

Circle the correct option: Overpaid/Owe by \$ [ ]

(D) (10) According to the *multiplier effect* in economics, when there is an injection of money to consumers in an economy, the consumers spend a certain proportion of it, then that amount recirculates through the economy and adds additional income, which comes back to the consumers and they spend the same percentage, etc. The process repeats *indefinitely* circulating additional money through the economy. Suppose the government cuts taxes by \$ 50 billion, thereby giving that much money back to consumers. If consumers save 10% of the money they get and spend the other 90%, what is the total additional spending circulated through the economy by the tax cut?

Amount = $\boxed{}$

(E) (10) Let $y(t)$ represent the population of a colony of tree frogs that is undergoing logistic growth following the differential equation $\frac{dy}{dt} = (.1)y(1 - \frac{y}{100})$, t in years. If the initial population is $y(0) = 10$, how long does it take for the population to reach 45?

Time to reach $y = 5$ is: $\boxed{}$