College of the Holy Cross, Spring 2018
Math 134 Midterm Exam 4
Friday, May 4

Your Name:

Instructions: For full credit, you must show all work on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values.

Please do not write in the space below

| Problem | Points/Poss |
| :--- | ---: |
| I | $/ 25$ |
| II | $/ 20$ |
| III | $/ 40$ |
| IV | $/ 15$ |
| Total | $/ 100$ |

I. The time $t$ (in seconds) between two successive incoming calls at a very busy consumer service call center is a random variable with pdf

$$
f(t)= \begin{cases}5 e^{-0.2 t} & \text { if } t \geq 0 \\ 0 & \text { if } t<0\end{cases}
$$

A. (5) Circle the integral that represents the probability that the time between successive calls is at least 1 second:

$$
\int_{0}^{1} 5 e^{-0.2 t} d t \quad \int_{1}^{\infty} 5 e^{-0.2 t} d t
$$

B. (10) Evaluate the integral you circled in part A.

$$
\text { Integral }=\square
$$

C. (10) The median time between calls is the time $m$ for which $P(t \leq m)=P(t \geq m)=$ $1 / 2$. Determine the median time between calls at this center.

$$
\text { median time }=\square
$$

II. Use the attached table of normal curve areas to answer the following questions.
A. (10) If $Z$ has a standard normal distribution, what is $P(-1.00<Z<1.34)$ ?

$$
\text { Probability }=\square
$$

B. (10) Bottles of cream soda have a stated volume of 20 fluid ounces, but the actual volume of the contents is normally distributed with mean 20.2 fluid oz. and $\mathrm{SD}=.4$ fluid oz. What is the probability that a randomly selected bottle contains more than 20 fluid oz. of soda?

$$
\text { Probability }=\square
$$

## III. (Differential Equations)

A. (20) Find the general solution of the differential equation

$$
\frac{d y}{d x}=\left(1+y^{2}\right) x^{3} \ln (x)
$$

$$
y=\square
$$

B. (5) Newton's Law of Heating/Cooling says that if an object is placed into a surrounding medium of constant temperature $A$, then the rate of change of the object's temperature is proportional to the difference between the object's temperature and the surrounding temperature. Circle the differential equation that corresponds to this statement:

$$
\frac{d y}{d t}=k(y-A) \quad \frac{d y}{d t}=\frac{k}{y-A}
$$

C. (15) A roasted chicken is taken out of the oven at a temperature of $190^{\circ} \mathrm{C}$ and left to cool on a counter in a room maintained at a constant temperature of $20^{\circ} \mathrm{C}$. After 10 minutes the temperature of the chicken has decreased to $100^{\circ} \mathrm{C}$. How long will it take for the temperature to reach $45^{\circ} \mathrm{C}$ ?

Time $=\square$
IV.
A. (5) What is the future value of $\$ 100,000$ at an interest rate of $5 \%$, compounded monthly for 15 years?

B. (5) Suppose you make monthly payments of $\$ 700$ for 15 years at $5 \%$, what is the future value of that stream of payments?

C. (5) Suppose you were making those payments from part B to a bank in return for a loan of $\$ 100,000$. At the end of the 15 years, would you still owe the bank, or would you have overpaid? Explain.

