College of the Holy Cross, Spring 2018
Math 134 Midterm Exam 3
Friday, April 13

Your Name:

Instructions: For full credit, you must show all work on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values.

Please do not write in the space below

| Problem | Points/Poss |
| :--- | ---: |
| I | $/ 30$ |
| II | $/ 20$ |
| III | $/ 40$ |
| IV | $/ 10$ |
| Total | $/ 100$ |

I. Compute each of the following integrals using the trigonometric reduction formulas (on separate sheet) and other methods as appropriate.
A. (15) $\int \sin ^{4}(5 x) d x$

$$
\text { Integral }=\square
$$

B. (15) $\int x \sec ^{4}\left(3 x^{2}\right) d x$

$$
\text { Integral }=\square
$$

II. (Trigonometric Substitutions)
A. (10) What trigonometric substitution would you use to evaluate

$$
\int \frac{x^{2}}{\sqrt{64+x^{2}}} d x ?
$$

Make the substitution and simplify so there is no square root in the resulting trigonometric integral.

$$
\text { Integral }=\square
$$

B. (10) Suppose you had used the trig substitution $x=3 \sin \theta$ and you integrated the resulting trigonometric integral to get

$$
\ln |\csc \theta-\cot \theta|+C
$$

What is the integral, expressed as a function of $x$ ?

$$
\text { Integral }=\square
$$

III. (Partial Fractions)
A. (5) To integrate $\frac{x^{4}+1}{x^{3}+7 x+1}$, what would be the first step? Carry the step out and give the rational function to which you would apply the partial fraction decomposition.
B. (10) To decompose $\frac{1}{(x+4)^{2}(x+1)^{3}\left(x^{2}+9\right)}$ into partial fractions, what would the form of the fractions be (leave coefficients as undetermined; do not try to solve for them).
C. (10) Determine the values $A, B, C$ making

$$
\frac{3 x+1}{x\left(x^{2}+4\right)}=\frac{A}{x}+\frac{B x+C}{x^{2}+4}
$$

$$
A=\square B=\square C=\square
$$

D. (15) You have decomposed a rational function $f(x)$ into partial fractions as

$$
f(x)=x^{2}+3 x+\frac{1}{(x+3)^{2}}+\frac{2}{(x+3)^{3}}+\frac{4 x+3}{x^{2}+1} .
$$

What is $\int f(x) d x$ ?

Integral $=\square$
IV. (Improper Integrals)
A. (5) Is there a finite area between the $x$-axis and the graph $y=\frac{1}{x^{2 / 3}}$ for $0<x \leq 1$ ? Set up the appropriate (improper) integral and determine if it converges.
B. (5) Does the solid of revolution obtained by rotating the region from part A about the $x$-axis have a finite volume or not? Why?

