Mathematics 136 – Calculus 2
Exam 1 – Review Sheet
February 14, 2020

General Information

As announced in the course syllabus, the first midterm exam of the semester will be given in class on Friday, February 21. There will be four or five questions (each one with several parts), similar in format to the sample exam questions below.

- You may use a scientific calculator for the exam which may have graphing capabilities.
- Use of cell phones, I-pods, and all other electronic devices besides a calculator *is not allowed* during the exam. Please leave such devices in your room or put them away in your backpack (make sure cell phones are turned off).

What will be covered

The exam will cover the material since the start of the semester – Problem Sets 1, 2, 3, including the following material from sections 5.1 through 5.7 and 7.1 through 7.3 of Rogawski and Adams:

1) Riemann sums and the definition of the definite integral.
2) The Fundamental Theorem of Calculus: Part 1: If $f(t)$ is continuous on $[a, b]$ and $F(x) = \int_a^x f(t)\ dt$, then $F'(x) = f(x)$. Part 2: (the "Evaluation Theorem") If $G(x)$ is an antiderivative of a continuous function $f(x)$ on $[a, b]$, then

$$\int_a^b f(x)\ dx = G(b) - G(a)$$

   Recall, though: our discussion of these topics in class was slightly different from the way they are presented in the text. I will not ask you about the way Rogawski and Adams prove the "Evaluation Theorem" in their Section 5.4, but I might ask you about the technique we used to prove the first part of the Fundamental Theorem (what Rogawski and Adams call the second part – see p. 295-296).
3) Antiderivatives (indefinite integrals) and basic antiderivative rules: All rules coming from basic derivative formulas: Know $\int x^n\ dx$, $\int e^x\ dx$, $\int \sin(x)\ dx$, $\int \cos(x)\ dx$, $\int \frac{1}{x^2+1}\ dx$, $\int \frac{1}{\sqrt{1-x^2}}\ dx$, and so forth, plus the sum, and constant multiple rules
4) Net change, the formula $\int_a^b f'(x)\ dx = f(b) - f(a)$ and its consequences and uses.
5) Integrals by u-substitution
6) Integration by parts and reduction formulas
7) Trigonometric integrals, using the short table of reduction formulas passed out in class on February 11. (Note: You do *not* need to memorize all of them; a copy of the formula sheet or a subset will be provided for your use during the exam.)
8) Integrals by trigonometric substitution.

There will be a review for the exam in class on Wednesday, February 19.

Review Problems

The Review Problems 1–30, and 39–118 at the end of Chapter 5 and 3–11 and 13–29 are good for preparation for this exam. It's not necessary to work all of them. But you should try a good selection and practice choosing a method. The odd-numbered problems have answers given in the back of the book so you can check your work.

Sample Exam Questions

Note: The actual exam will be **considerably** *shorter than the following list of questions. The purpose here is just to give an idea of the range of different topics that will be covered and how questions might be posed.*

I.

A) Evaluate the L_4, R_4, and M_4 Riemann sums for $f(x) = x^2 - 2x$ for $0 \le x \le 1$ (Note: you are using $N = 4$ each time).

B) In part A), one of your values is definitely larger than the actual value of $\int_0^1 x^2 - 2x\,dx$ and one is definitely smaller than the integral. Which is which? (Answer without calculating the value of the integral and then check your work.)

C) The following limit represents a definite integral:

$$\lim_{N\to\infty} \sum_{j=1}^{N} \frac{\cos\left(\frac{j\pi}{N}\right)}{\left(\frac{j\pi}{N}\right)^2 + 1} \cdot \frac{\pi}{N}$$

What is the integral?

II.

A) A particle moves along a straight-line path with velocity given by $v(t) = \sqrt{t+5}$ (meters per second) at time t (in seconds). Estimate the total distance traveled between $t = 0$ and $t = 10$ by using the L_5 (left-hand sum) approximation.

B) Express the exact value of the total distance traveled as a definite integral and evaluate it using the Fundamental Theorem of Calculus.

III.

A) Compute $\lim_{N\to\infty} \sum_{i=1}^{N} \left(\frac{5i}{N}\right)^2 \frac{5}{N}$ using the power sum formulas from page 265. (Note: If I ask a question like this, I would give those formulas – you don't need to memorize them.)

B) The limit in part (A) computes a definite integral. Using the Fundamental Theorem of Calculus (part I), check your work in part (A).

IV. Let

$$f(x) = \begin{cases} 1 & \text{if } 0 \le x \le 3 \\ x - 2 & \text{if } 3 \le x \le 5 \\ 13 - 2x & \text{if } 5 \le x \le 8 \end{cases}$$

A) Sketch the graph $y = f(x)$. In the rest of the parts, $F(x) = \int_0^x f(t)\, dt$, where f is the function from part A.

B) Compute $F(1), F(2), F(3), F(4), F(5), F(6), F(7), F(8)$ given the information in the graph of f.

C) Are there any critical points of F (places where $F'(x) = 0$)? If so, find them and say whether they are local maxima, local minima, or neither. If not, say why not.

D) How is the graph of $F(x)$ related to the graph of

$$G(x) = \int_2^x f(t)\, dt?$$

V. Find the *derivatives* of the following functions:

A) $f(x) = \int_0^x \sin(t)/t\, dt$.

B) $g(x) = \int_5^{x^3} \tan^4(t)\, dt$.

C) $h(x) = \int_{-3x}^{5x} e^{t^2} \sin(t)\, dt$.

VI. Note: I will probably ask some questions in this form, indicating a method to use. But you should also be prepared for others as in the next question where you need to choose which method applies.

A) Compute $\int 5x^4 - 3\sqrt{x} + e^x + \frac{2}{x}\, dx$ using basic integration formulas.

B) Apply a u-substitution to compute $\int x(4x^2 - 3)^{3/5}\, dx$

C) Integrate by parts to compute $\int x^2 e^{6x}\, dx$

D) Integrate with an appropriate u-substitution:

$$\int \frac{t^2 + 1}{t^3 + 3t + 3}\, dt$$

(Hint: How does the bottom relate to the top?)

E) Compute $\int x^4 \ln(3x)\, dx$ using integration by parts.

F) Find $\int \sin^4(3x)\, dx$ using our trigonometric reduction formulas.

VII. Compute each of the integrals below. You must show all work for full credit.

A)

$$\int \frac{e^{\sqrt{\sin(x)}} \cos(x)}{\sqrt{\sin(x)}}\, dx$$

B)

$$\int \cos^2(2x) \sin^2(2x)\, dx$$

C)

$$\int \frac{x^2\ dx}{\sqrt{16 - x^2}}$$

D)

$$\int \frac{x + 2}{x^2 + 1}\ dx$$

(Split into two fractions and integrate each separately.)