MATH 136 – Calculus 2
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Background If a solid extends along the x-axis between a and b and the area of the cross-section by a plane $x = \text{const}$ is given by some function $A(x)$ for all x, then *Cavalieri's Principle* (named after the Italian Renaissance mathematician Bonaventura Cavalieri, 1598-1647) says that

$$\text{Volume} = \int_a^b A(x)\, dx$$

Questions

1. A circular cone extends from $x = 0$ to $x = 4$ along the x-axis. The cross-section in each plane $x = \text{constant}$ is a circle whose radius increases linearly from $r = 0$ at $x = 0$ to $r = 5$ by the time x reaches 4.
   (a) Write the radius r as a function of x on the interval 0 to 4.
   (b) Write the area of the circular cross-section as a function of x.
   (c) Find the volume by applying the Cavalieri Principle equation above.
   (d) Check your result with the formula for the volume of a cone from high school geometry.
2. A solid extends along the x-axis from $x = -3$ to $x = 3$. The cross-section in each plane $x = \text{constant}$ is a semicircle with radius e^x. Find the volume of the solid using Cavalieri's Principle.
3. A solid extends from $x = 0$ to $x = 10$ along the x-axis. The cross-section at x is a square with side $\sqrt[4]{25 + x^2}$. Set up the integral to compute the volume using Cavalieri's Principle. Evaluate the integral using a trigonometric substitution or a table of integrals.
4. In the previous problem, would it make an difference if the squares all had center point along the x-axis or if all the squares had one corner at a point on the x-axis?