MATH 136 – Calculus 2
Practice Day on u-substitution
February 4, 2020

Background

Recall that u-substitution is the name given to the indefinite integration method coming from the Chain Rule for derivatives. In its most basic form, it just says that if F is an antiderivative of f, then

$$\int f(u(x))\frac{du}{dx}\ dx = F(u(x)) + C.$$

Note that $\frac{du}{dx} = u'(x)$ appears in the integral on the left. If that term were not there, then this method would not "work." The method of u-substitution is a *change of variables* method, where we essentially rewrite the integral above as

$$\int f(u)\ du = F(u) + C$$

involving functions of only the new variable u. The goal is to use the known antiderivative $F(u)$ for the function $f(u)$ as a function of u.

The most important aspects to master here are

- recognizing good candidates for the u to substitute for
- always remembering to compute $du = \frac{du}{dx}\ dx$ and to match that with the rest of the integral other than the $f(u)$ part,
- using the other basic derivative rules to find F from f.

Questions

For each problem,

(i) find a candidate u,

(ii) compute $du = \frac{du}{dx}\ dx$

(iii) see whether the rest of the integrand can be matched with du, possibly up to a constant multiple (if not, then you might need to try a different u or a different method entirely),

(iv) finish the integration.

1. $\int x^2 \cos(x^3)\ dx$

   Solution: Let $u = x^3$, then $du = 3x^2\ dx$, so the $x^2 = \frac{1}{3}du$ and the integral becomes

   $$\int \frac{1}{3}\cos(u)\ du = \frac{1}{3}\sin(u) + C = \frac{1}{3}\sin(x^3) + C$$

2. $\int (\sqrt{x}+1)^4 \cdot \frac{1}{\sqrt{x}}\ dx$

   Solution: Let $u = \sqrt{x}+1 = x^{1/2}+1$. Then $du = \frac{1}{2}x^{-1/2}\ dx = \frac{dx}{2\sqrt{x}}$. In terms of u, the integrand becomes:

   $$\int u^4 \cdot 2\ du = \frac{2}{5}u^5 + C = \frac{2}{5}(\sqrt{x}+1)^5 + C$$

   $$\int \frac{x+4}{x^2+8x+9}\ dx$$

3. *Solution:* Here we have to recognize that if $u = x^2 + 8x + 9$, then $du = (2x+8)\ dx = 2(x+4)\ dx$. This is twice the numerator of the fraction so the form is

   $$\frac{1}{2}\int \frac{1}{u}\ du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|x^2+8x+9| + C$$

4. $\int e^{\tan(x)}\sec^2(x)\ dx$

   Solution: Let $u = \tan(x)$. Then $du = \sec^2(x)\ dx$ on the nose. So we have $\int e^u\ du = e^u + C = e^{\tan(x)} + C$.

5. $\int \frac{x}{\sqrt{1-x^4}}\ dx$

   Solution: This looks like an inverse sine integral. To verify that, notice that $x^4 = (x^2)^2$. So with $u = x^2$, we have $du = 2x\ dx$, and the integrand is

   $$\frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} = \frac{1}{2}\sin^{-1}(u) + C = \frac{1}{2}\sin^{-1}(x^2) + C$$