MATH 136 – Calculus 2
Second Practice Day on u-substitution
February 5, 2020

Background

Here are a few additional integration formulas that can be used in conjunction with u-substitution:

- If $b > 0$, then $\int b^x \, dx = \frac{b^x}{\ln(b)} + C$ (This follows by combining the derivative rule for e^u by the chain rule with the formula $b^x = (e^{\ln(b)})^x = e^{x\ln(b)}$.)
- $\int \frac{dx}{|x|\sqrt{x^2-1}} = \sec^{-1}(x) + C$ (follows from the derivative rule

$$\frac{d}{dx}\sec^{-1}(x) = \frac{1}{|x|\sqrt{x^2-1}}$$

Questions

Find the following integrals using the formulas above and u-substitution as needed:

1. $\int 5^{\cos(x)} \sin(x) \, dx$

   Solution: Use the rule for b^x with $b = 5$ and $u = \cos(x)$. Then $du = -\sin(x) \, dx$, so we have

$$-\int 5^u \, du = -\frac{5^u}{\ln(5)} + C = -\frac{5^{\cos(x)}}{\ln(5)} + C$$

2. $\int \frac{dx}{(x+8)\ln(2x+16)}$

   Solution: This one is slightly tricky. We need to notice that if $u = \ln(2x+16)$, then $du = \frac{1}{2x+16} \cdot 2 \, dx = \frac{1}{x+8} \, dx$. This means that the integral is

$$\int \frac{1}{u} \, du = \ln|u| + C = \ln|\ln(2x+16)| + C$$

3. $\displaystyle\int \frac{dx}{x\sqrt{49x^2-1}}$ (Take $x > 0$.)

Solution: Let $u = 7x$. Then $du = 7\ dx$. The integral becomes

$$\int \frac{du/7}{(u/7)\sqrt{u^2-1}} = \int \frac{du}{u\sqrt{u^2-1}} = \sec^{-1}(u) + C = \sec^{-1}(7x) + C$$

4. $\displaystyle\int x^2\sqrt{x+4}\ dx$ (Hint: Let $u = x + 4$)

Solution: Using the Hint, $du = dx$, $x = u - 4$, and the integrand becomes

$$\begin{aligned}\int (u-4)^2\sqrt{u}\ du &= \int u^{5/2} - 8u^{3/2} + 16u^{1/2}\ du \\ &= \frac{2}{7}u^{7/2} - \frac{16}{5}u^{5/2} + \frac{32}{3}u^{3/2} + C \\ &= \frac{2}{7}(x+4)^{7/2} - \frac{16}{5}(x+4)^{5/2} + \frac{32}{3}(x+4)^{3/2} + C\end{aligned}$$

5. $\displaystyle\int_0^1 4^{3x}\ dx$

Solution: Let $u = 3x$, then converting everything to equivalent expressions in u (including the limits of integration), the integral is

$$\int_{u=0}^{u=3} \frac{1}{3}4^u\ du = \frac{1}{3\ln(4)}4^u\Big|_{u=0}^{u=3} = \frac{1}{3\ln(4)}(4^3-1) = \frac{21}{\ln(4)}.$$