MATH 136 – Calculus 2
Practice Day on u-substitution
February 4, 2020

Background

Recall that u-substitution is the name given to the indefinite integration method coming from the Chain Rule for derivatives. In its most basic form, it just says that if F is an antiderivative of f, then

$$\int f(u(x))\frac{du}{dx}\;dx = F(u(x)) + C.$$

Note that $\frac{du}{dx} = u'(x)$ appears in the integral on the left. If that term were not there, then this method would not "work." The method of u-substitution is a *change of variables* method, where we essentially rewrite the integral above as

$$\int f(u)\;du = F(u) + C$$

involving functions of only the new variable u. The goal is to use the known antiderivative $F(u)$ for the function $f(u)$ as a function of u.

The most important aspects to master here are

- recognizing good candidates for the u to substitute for
- always remembering to compute $du = \frac{du}{dx}\;dx$ and to match that with the rest of the integral other than the $f(u)$ part,
- using the other basic derivative rules to find F from f.

Questions

For each problem,

(i) find a candidate u,

(ii) compute $du = \frac{du}{dx}\;dx$

(iii) see whether the rest of the integrand can be matched with du, possibly up to a constant multiple (if not, then you might need to try a different u or a different method entirely),

(iv) finish the integration.

1. $\int x^2 \cos(x^3)\ dx$

2. $\int (\sqrt{x}+1)^4 \cdot \frac{1}{\sqrt{x}}\ dx$

3. $\int \frac{x+4}{x^2+8x+9}\ dx$

4. $\int e^{\tan(x)} \sec^2(x)\ dx$

5. $\int \frac{x}{\sqrt{1-x^4}}\ dx$