MATH 136 – Calculus 2
Practice Day on Trigonometric Substitution Integrals
February 14, 2020

Background

Recall that we saw some first examples of the *trigonometric substitution* method last time. The basic outline of this method is that for integrals involving

- $\sqrt{a^2 - x^2}$, we let $x = a \sin\theta$ and $dx = a\cos\theta \, d\theta$
- $\sqrt{a^2 + x^2}$, we let $x = a \tan\theta$ and $dx = a\sec^2\theta \, d\theta$
- $\sqrt{x^2 - a^2}$, we let $x = a \sec\theta$ and $dx = a\sec\theta\tan\theta \, d\theta$

We simplify, then apply our trig reduction formulas from last week. The last step is to convert back to functions of x using the reference triangle corresponding to the substition used:

- For the $x = a\sin\theta$ substitution, put x on opposite side, a on hypotenuse, then $(adj) = \sqrt{a^2 - x^2}$, so you can read off any trig function of θ from the triangle and $\theta = \sin^{-1}(x/a)$
- For the $x = a\tan\theta$ substitution, put x on opposite side, a on adjacent, then $(hyp) = \sqrt{x^2 + a^2}$, so you can read off any trig function of θ from the triangle and $\theta = \tan^{-1}(x/a)$
- For the $x = a\sec\theta$ substitution, put x on hypotenuse, a on adjacent, then $(opp) = \sqrt{x^2 - a^2}$, so you can read off any trig function of θ from the triangle and $\theta = \sec^{-1}(x/a)$.

Questions

(A) Using the appropriate trigonometric substitution compute

$$\int \frac{1}{\sqrt{x^2 + 25}} \, dx$$

(B) Using the appropriate trigonometric substitution compute

$$\int \frac{1}{(36 - x^2)^{3/2}} \, dx$$

(C) Using the appropriate trigonometric substitution compute

$$\int \frac{1}{\sqrt{x^2 - 16}}\, dx$$

(D) You want to divide a circular pizza with radius 9in (say with outer crust along the circle $x^2 + y^2 = 81$) into three exactly equal pieces with a knife or pizza cutter. One way, of course, is to divide it into three sectors each with angle exactly $2\pi/3$ by cutting from the center to the outer edge. But another (easier?) way would be to cut the pizza with two parallel strokes, say at $x = \pm a$ so that the three strips $-9 \leq x \leq -a$, $-a \leq x \leq a$, and $a \leq x \leq 9$ all have the same area. Set up an integral with a in the limits of integration, evaluate it using an appropriate trigonometric substitution, and find an equation to solve for the a giving three strips of exactly equal area. Find an approximation for a by using a graphing calculator.