MATH 136 – Calculus 2
Practice Day on Trigonometric Integrals
February 11, 2020

Background

Our approach to trigonometric integrals will be based in systematic application of *reduction formulas* like the ones we saw yesterday. When you do these and exams, the formulas on the sheet I distributed *will be provided* for your use in table format. You will be responsible for deciding which formula applies and for applying it correctly. Here are some practice problems.

Questions

For each of the following integrals, decide which of the reduction formula applies, determine the appropriate n (and m in the last two). Then apply the formula and complete the computation.

1. $\displaystyle\int \sin^2(x)\ dx$

   Answer: Use SC1:
   $$\frac{x}{2} - \frac{\sin(x)\cos(x)}{2} + C$$

2. $\displaystyle\int \sec^3(x)\ dx$

   Answer: Use ST3 and ST4:
   $$\frac{\sec(x)\tan(x)}{2} + \frac{1}{2}\ln|\sec(x) + \tan(x)| + C$$

3. $\displaystyle\int \cos^4(3x)\ dx$

   Answer: Use SC2 twice:
   $$\frac{3x}{8} + \frac{\cos^3(3x)\sin(3x)}{12} + \frac{\cos(3x)\sin(3x)}{8} + C$$

4. $\displaystyle\int \tan^2(4x)\sec^3(4x)\ dx$ (Use $\tan^2(u) = 1 + \sec^2(u)$, then ST3.)

Answer: Using the identity yields the equivalent sum of two integrals:

$$\int \sec^3(4x)\ dx + \int \sec^5(4x)\ dx$$

Using the reduction formulas and simplifying yields:

$$\frac{\sec^3(4x)\tan(4x)}{16} + \frac{7\sec(4x)\tan(4x)}{32} + \frac{7}{32}\ln|\sec(4x)+\tan(4x)| + C.$$

5. $\int \tan^3(5x)\sec^5(5x)\ dx$ (Convert to powers of $\sin(u)$ and $\cos(u)$.)

Answer: This can be done using SC4 or more cleverly by rewriting $\sin^3(5x)$ as $\sin(5x)\cdot\sin^2(5x) = \sin(5x)\cdot(1-\cos^2(5x))$:

$$\begin{aligned}\int \frac{\sin^3(5x)}{\cos^8(5x)}\ dx &= \int \frac{\sin(5x)}{\cos^8(5x)}\ dx - \int \frac{\sin(5x)}{\cos^6(5x)}\ dx \\ &= \frac{-1}{35}\cos^{-7}(5x) + \frac{1}{25}\cos^{-5}(5x) + C\end{aligned}$$