MATH 136 – Calculus 2
Practice Day on Consequences of FTC, "Part I"
January 31, 2020

Background

Recall that the "FTC, part I" says: If f is continuous on $[a, b]$ and F is any function satisfying $\frac{d}{dx}F(x) = F'(x) = f(x)$ for all x in $[a, b]$ (called an *antiderivative* of f), then

$$\int_a^b f(x)\, dx = F(b) - F(a).$$

We will use the *shortcut notation*: $F(x)|_a^b$ for this difference of values at the limits of integration:

$$F(x)|_a^b = F(b) - F(a).$$

Note that you always put the value $F(b)$ at the *upper limit* first and then subtract the value at the lower limit, $F(a)$. This means that we have a "shortcut method" for computing $\int_a^b f(x)\, dx$ as long as we have or can find a suitable *antiderivative* F – a function with $F'(x) = f(x)$ for the function $f(x)$ we are integrating.

Questions

(1) Verify that

$$\frac{d}{dx}(e^{x^2}) = 2xe^{x^2}.$$

(2) Use part (1) to evaluate:

$$\int_0^1 2xe^{x^2}\, dx$$

(3) Verify that

$$\frac{d}{dx}(\ln(x^2 + 5x + 6)) = \frac{2x + 5}{x^2 + 5x + 6}.$$

(4) Use part (4) to evaluate:

$$\int_1^2 \frac{2x + 5}{x^2 + 5x + 6}\, dx.$$

(5) Verify that

$$\frac{d}{dx}\left(\frac{1}{2}\left(x+\sin(x)\cos(x)\right)\right)=\cos^2(x)$$

(You'll need a trig identity here – ask if you don't recall identities using $\sin^2(x)$ and $\cos^2(x)$!)

(6) Use part (5) to evaluate:

$$\int_0^{\pi/2} \cos^2(x)\ dx.$$