MATH 136 – Calculus 2
Practice Day on Consequences of FTC, "Part II"
January 29, 2020

Background

Recall that the "FTC, part II" says: If f is continuous on $[a, b]$, then

$$\frac{d}{dx}\int_a^x f(t)\ dt = f(x)$$

for all x in $[a, b]$. (Note: at the endpoints $x = a$ and $x = b$, we may be talking only about "one-sided derivatives" if f is not defined for $x < a$ or $x > b$.)

Questions

(0) Look at the function $f(t)$ plotted on the back. What is $\frac{d}{dx}\int_0^x f(t)\ dt$ at $x = 3$?

(1) What is

$$\frac{d}{dx}\int_3^x \sqrt[3]{t^5 + 3t + 1}\ dt?$$

(2) Same question for

$$\frac{d}{dx}\int_0^x \frac{\tan(t)}{t^3 + 2}\ dt$$

(3) What is

$$\frac{d}{dx}\int_x^4 e^{4t^2+7}\ dt?$$

(4) What about

$$\frac{d}{dx}\int_0^{x^2} \sqrt{t^2 + 1}\ dt?$$

(Use the chain rule.)

(5) What is

$$\frac{d}{dx}\int_{-x^3}^{x^4} \frac{t}{t+1}$$

(You'll need to use the interval union property, the interchange of limits property and the chain rule here!)

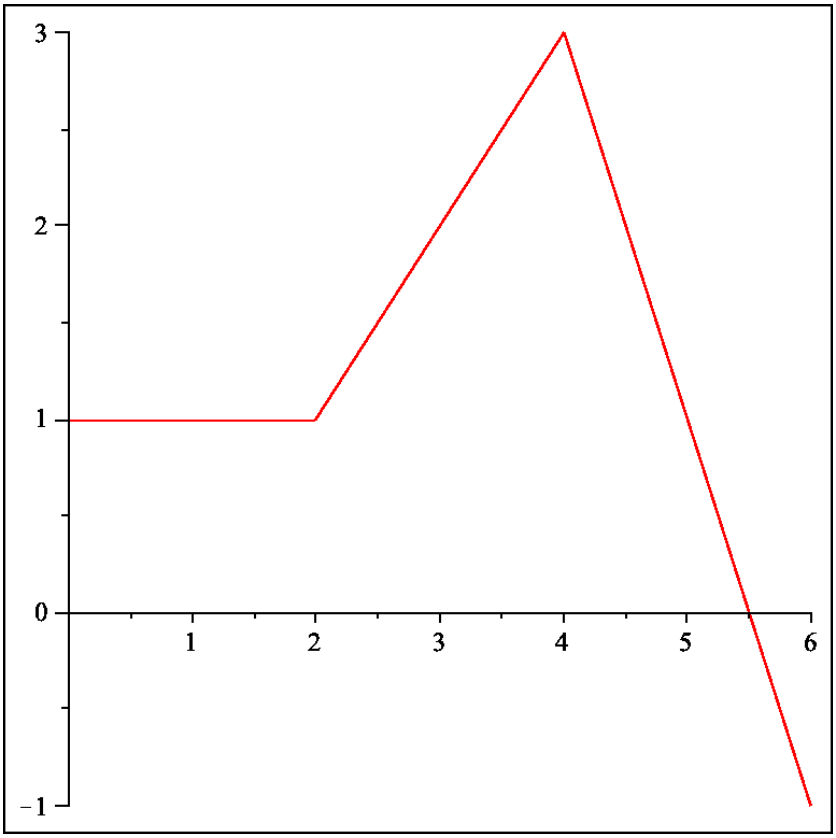

Figure 1: The region

(6) Finally, suppose someone asked you for a function whose derivative was $f(x) = \dfrac{x^2}{x^4 + x + 1}$. How could you "cook one up" using what we have talked about? (In case you are worried about possible discontinuities from vertical asymptotes, you don't need to: this function f is continuous for all real x – the denominator is never zero.)