MATH 136 – Calculus 2
Practice Day on Properties of $\int_a^b f(x)\,dx$
January 27, 2020

Background

We have now introduced the definite integral $\int_a^b f(x)\,dx$ and discussed:

- the interpretation of of this number as a *signed area*,
- the *interval additivity property*: If f is integrable on $[a,b]$ and $a \leq c \leq b$, then
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$
- In one video for today, we also saw the *comparison property*: If f is integrable on $[a,b]$ and $m \leq f(x) \leq M$ for all x in $[a,b]$, then
$$m \cdot (b-a) \leq \int_a^b f(x)\,dx \leq M \cdot (b-a).$$

Questions Let $f(x)$ be the function on the interval $[0,6]$ plotted on the back. Thinking of the definite integral as *signed area*, answer these questions.

(1) What are $\int_0^4 f(x)\,dx$, $\int_4^6 f(x)\,dx$, $\int_0^6 f(x)\,dx$? How can you get the last one using the first two? $\int_0^6 f(x)\,dx = \int_0^4 f(x)\,dx + \int_4^6 f(x)\,dx$.

(2) What is $\int_5^6 f(x)\,dx$? How can you tell?

(3) Let m be the minimum value of $f(x)$ on $[3,5]$ and let M be the maximum value of $f(x)$ on $[3,5]$ (use the information from the graph). Find m, M and verify that
$$m \cdot (5-3) \leq \int_3^5 f(x)\,dx \leq M \cdot (5-3)$$

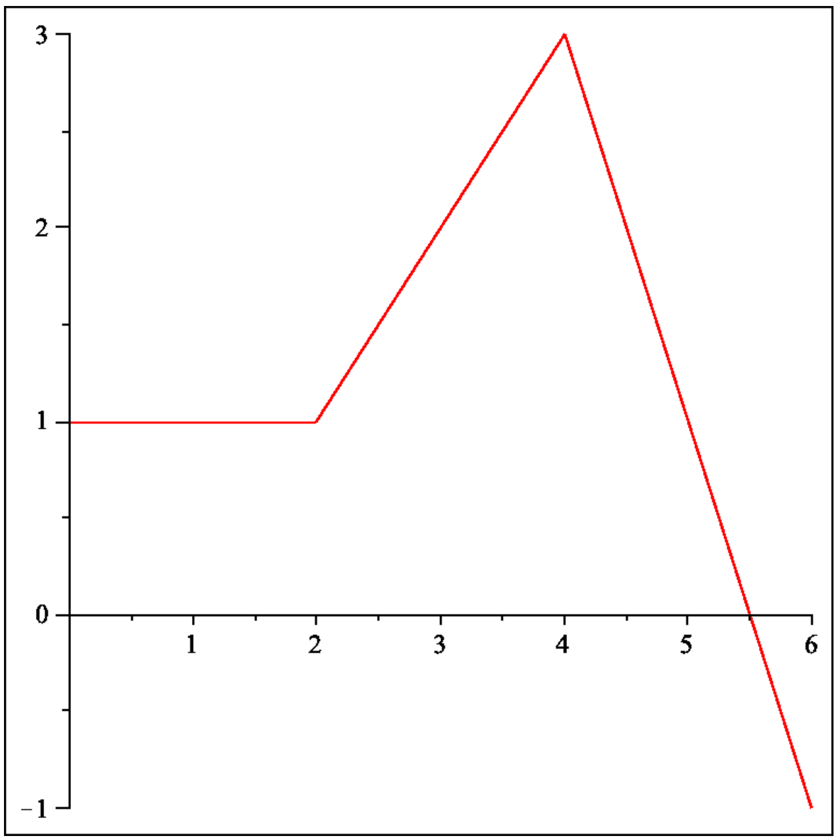

Figure 1: The region