MATH 136 – Calculus 2
Practice Day on the L_N, R_N, M_N Riemann Sums
January 24, 2020

Background

We have now introduced the L_N, R_N, M_N sum approximations to the area between the graph $y = f(x)$ and the x-axis, for x in $[a, b]$. All three sums are based on a *partition*, or subdividision, of the interval $[a, b]$ into some given number N of smaller intervals of width $\Delta x = \frac{b-a}{N}$. The endpoints of the smaller intervals are then given by

$$x_j = a + j\Delta x, \quad \text{for} \quad j = 0, 1, 2, \ldots, N.$$

The midpoint of the jth subinterval is

$$m_j = \frac{x_{j-1} + x_j}{2}.$$

Then we have

- using the left endpoints: $L_N = \sum_{j=1}^{N} f(x_{j-1})\Delta x$
- using the right endpoints: $R_N = \sum_{j=1}^{N} f(x_j)\Delta x$
- using the midpoints: $M_N = \sum_{j=1}^{N} f(m_j)\Delta x = \sum_{j=1}^{N} f\left(\frac{x_{j-1}+x_j}{2}\right)\Delta x$.

These are all special cases of the more general *Riemann sums*, named for the German Georg Friedrich Bernhard Riemann (1826 - 1866), who was one of the most celebrated and original mathematicians of the 19th century.

Questions

(A) Let $f(x) = x^2 + 3x + 1$ on the interval $[a, b] = [2, 4]$.

(1) Take $N = 3$ and compute the L_3, R_3, M_3 sums for this function.

(2) With the same function and the same interval, now take $N = 6$ and compute the L_6, R_6, M_6 sums for this function.

(3) Which is closer to the true area under the graph $y = x^2 + 3x + 1$ on $[2, 4]$, L_3 or L_6? Explain by drawing the picture of the graph and the approximating rectangles.

(4) Similarly, which is closer to the true area under the graph, R_3 or R_6?

(5) Of the six numbers you computed in parts (1) and (2), which do you think is *the closest* to the true area under the graph? Explain.

(B) Repeat parts (1) and (2) of question A for $f(x) = \sin(x)$ on the interval $[a, b] = [0, \pi]$. Explain why the answers to parts (3), (4), and (5) are not as easy to see in this case(!)