MATH 136 – Calculus 2
Solutions for Practice on Approximating Areas
January 22, 2020

Background

We have discussed the idea of approximating the area between a graph $y = f(x) \geq 0$ and the x-axis for $a \leq b \leq x$ by using sums of areas of rectangles and worked out a first example. To practice using these ideas you will work through another similar example using a different function, interval, and subdivision. Here's the description of the region we want in formulas: The region between $y = x^2 - 4x + 6$ and the x-axis, for $1 \leq x \leq 2$.

Questions

1. Sketch the region (you may want to complete the square in $x^2 - 4x + 6$ to do this). Check your work by looking at the figure on the back of this page.

   Answer: Completing the square, $x^2 - 4x + 6 = (x-2)^2 + 2$. So the graph is a parabola opening up, with the vertex at the point $(2, 2)$. See graphs on back, which show the segment of this parabola for x in $[1, 2]$.

2. To subdivide the x-interval $[1, 2]$ into $N = 5$ equal smaller intervals, what Δx should we use? What are the endpoints $x_0, x_1, x_2, x_3, x_4, x_5$ of the intervals?

   Answer: $\Delta x = \frac{2-1}{5} = \frac{1}{5} = 0.2$. So the end points are $x_i = 1 + i\Delta x = 1 + (.2)i$, or

   $$x_0 = 1, x_1 = 1.2, x_2 = 1.4, x_3 = 1.6, x_4 = 1.8, x_5 = 2$$

3. Make rectangles using the value of the function at the *left endpoint* of each smaller interval to get the heights. Sketch these rectangles together with the graph.

   Answer: See Figure 1 on back.

4. Write the sum giving the areas of these rectangles using summation notation, and compute the numerical value.

   Answer: The sum is

   $$\sum_{i=0}^{4} (x_i^2 - 4x_i + 6) \cdot \Delta x = \sum_{i=0}^{4} ((1 + .2i)^2 - 4(1 + .2i) + 6)(.2)$$

   The numerical value is 2.44.

5. Now make rectangles using the value of the function at the *right endpoint* of each smaller interval to get the heights. Sketch these rectangles together with the graph.

   Answer: See Figure 2 on back.

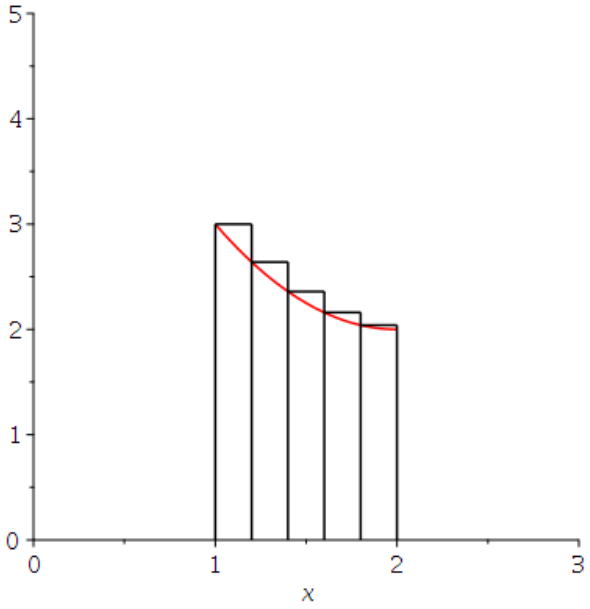

Figure 1: The left endpoint rectangles

6. Write the sum giving the areas of these rectangles using summation notation, and compute the numerical value.

   Answer: The sum is

$$\sum_{i=1}^{5}(x_i^2 - 4x_i + 6) \cdot \Delta x = \sum_{i=1}^{5}((1 + .2i)^2 - 4(1 + .2i) + 6)(.2)$$

   The numerical value is 2.24.

7. What can you say about the area of the region from your computations?

   Answer: The function $f(x) = x^2 - 4x + 6$ is decreasing on the whole interval $[1, 2]$. So the left-endpoint rectangles completely cover the area under the graph, while the right-endpoint rectangles "miss" some of the area under the graph. This says

$$2.24 < \text{Area} < 2.44$$

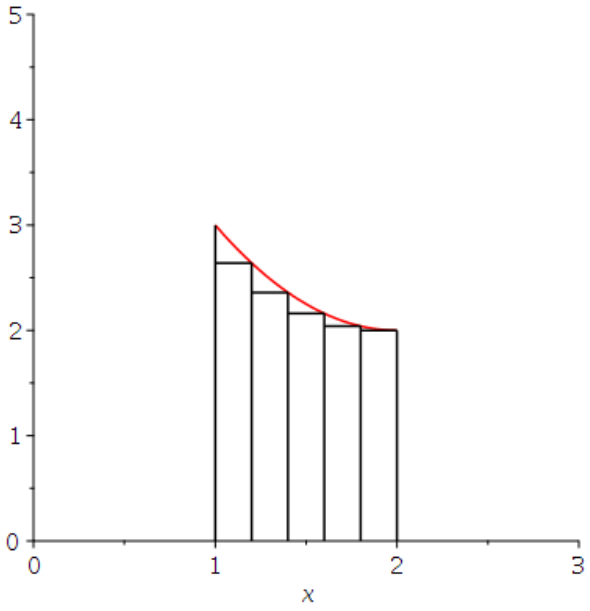

Figure 2: The right endpoint rectangles