MATH 136 – Calculus 2
Normal Probabilities and Tabulated Values
March 18, 2020

*Background*

The standard normal pdf is the function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

The factor $1/\sqrt{2\pi}$ makes the total area equal to 1 as we require for a pdf, but this is not easy to show given what we have learned to this point. Probabilities for a standard normal random variable (i.e. normal distribution with $\mu = 0$, $\sigma = 1$) are given in the table on the accompanying sheet.

In today's problems, you will learn more about normal pdfs and practice using the table to answer questions about normally distributed quantities.

*Questions*

A) The standard normal pdf is a function that has *no elementary antiderivative*. This means that the entries in the table on the back were computed by approximate numerical integration techniques. Approximate the area given by

$$\int_0^1 \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$

using a midpoint Riemann sum with $N = 5$. How close is your value to the table value for $z = 1.0$? What would you need to do to get a more accurate value?

B) The standard normal pdf above is just one of a 2-parameter family of normal pdfs give by the formula

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

The parameter $\mu$ shifts the center of the distribution, and the $\sigma$ controls the width of the peak. Show using the subsitution $u = \frac{x-\mu}{\sigma}$ that

$$\int_a^b \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2}\, dx = \int_{(a-\mu)/\sigma}^{(b-\mu)/\sigma} \frac{1}{\sqrt{2\pi}} e^{-u^2/2}\, du$$

In other words, areas under *any normal pdf* graph can be computed from the table for the standard normal by taking the original limits of integration $a, b$ and "standardizing" to the new limits

$$\frac{a-\mu}{\sigma}, \quad \frac{b-\mu}{\sigma}$$

for the standard normal.

C) Let $Z$ be a standard normal.

1) Find $P(-2.13 < Z < -0.56)$
2) Find $c$ such that $P(Z > c) = .05$

D) Let $Y$ be normally distributed with mean $\mu 6$ and $\sigma = 2$. Find

1) $P(6 < Y < 7)$
2) $P(7 < Y < 8)$

E) Small packages of Oyster brand chowder crackers have a stated label weight of 45 grams. The actual weights of the packages are normally distributed with mean $\mu = 46.3$ grams and $\sigma = .7$ grams. Let $Y$ be the weight of a single package selected at random from the production line. What is the probability $P(Y > 46)$?