MATH 136 – Calculus 2
First Practice on Improper Integrals
February 25, 2020

Background

Whenever $a = -\infty$ or $b = +\infty$ or both, *or* the function $f(x)$ has a discontinuity in the interval $[a, b]$, the integral $\displaystyle\int_a^b f(x)\, dx$ is said to be an *improper integral*. Improper integrals are always handled by *taking limits of "ordinary" integrals*.

- We say $\int_a^\infty f(x)\, dx$ *converges* if the limit

  $$\lim_{b\to\infty} \int_a^b f(x)\, dx$$

  exists and we say the integral *diverges* otherwise.

- We say $\int_{-\infty}^b f(x)\, dx$ *converges* if the limit

  $$\lim_{a\to-\infty} \int_a^b f(x)\, dx$$

  exists and we say the integral *diverges* otherwise.

- We say $\int_{-\infty}^\infty f(x)\, dx$ *converges* if, splitting at any finite value c (often at $c = 0$), *both of the limits* here:

  $$\lim_{a\to-\infty} \int_a^c f(x)\, dx + \lim_{b\to+\infty} \int_c^b f(x)\, dx$$

  exist and we say the integral *diverges* otherwise. (Note this says that if either one of the two limits does not exist, then the integral diverges.)

- If f is discontinuous at a, then we say $\int_a^b f(x)\, dx$ *converges* if the limit

  $$\lim_{c\to a^+} \int_c^b f(x)\, dx$$

  exists and we say the integral *diverges* otherwise.

- If f is discontinuous at b, then we say $\int_a^b f(x)\,dx$ *converges* if the limit

$$\lim_{c \to b^-} \int_a^c f(x)\,dx$$

exists and we say the integral *diverges* otherwise.

- If f is discontinuous at d in (a, b), then we say $\int_a^b f(x)\,dx$ *converges* if *both of the limits* here:

$$\lim_{c \to d^-} \int_a^c f(x)\,dx + \lim_{c \to d^+} \int_c^b f(x)\,dx$$

exist and we say the integral *diverges* otherwise. (Note this says that if either one of the two limits does not exist, then the integral diverges.)

Questions

For each of the following,

(i) Explain why the integral is improper.

(ii) Set up the appropriate limit(s) and evaluate.

(iii) Say whether the integral converges or diverges.

1. $\displaystyle\int_{-\infty}^{2} e^{10x}\,dx$

2. $\displaystyle\int_{4}^{\infty} \frac{1}{x^2+16}\,dx$

3. $\displaystyle\int_{-\infty}^{\infty} \frac{1}{x^2+16}\,dx$

4. $\displaystyle\int_{1}^{2} \frac{1}{\sqrt[3]{x-1}}\,dx$

5. $\displaystyle\int_{0}^{2} \frac{1}{x^2-5x+6}\,dx$

6. $\displaystyle\int_{1}^{3} \frac{1}{x^2-4x+4}\,dx$