MATH 136 – Calculus 2
Practice on Separable Differential Equations and Applications
March 20, 2018

Background

A differential equation is a relation between derivative(s) of an unknown function and other known functions. A differential equation often serves as a *mathematical model* for how some quantity is evolving with respect to time. For example, Newton's Law of Cooling (and Heating) says: *if an object is placed into a surrounding medium held at constant temperature A, then the object's temperature changes at a rate proportional to the difference between its temperature and the ambient temperature.* If T is the object's temperature, this statement is equivalent to the differential equation

$$\frac{dT}{dt} = k(T - A)$$

for some constant k.

Any differential equation of the form

$$\frac{dy}{dx} = g(y) \cdot h(x)$$

is called a "separable" equation and can be solved by

- *separating the variables* to the form

$$\frac{dy}{g(y)} = h(x)\ dx$$

- *integrating on both sides* (on the left, treat the variable of integration as y, not x; this can be justified by the substitution method for integration):

$$\int \frac{dy}{g(y)} = \int h(x)\ dx$$

- *solving the resulting equation for y*

(A) To practice, solve the following separable equations:

(1)

$$\frac{dy}{dx} = \frac{y}{x^2 + 1}$$

(2)

$$\frac{dy}{dx} = x^3y^2 + y^2$$

(Hint: factor on the right, then you can separate variables)

(B) Solve the following Newton's Law of Cooling problem: A hot cup of coffee is poured at time $t = 0$ with the temperature being 80° C. The cup is placed on a desk in a room with temperature maintained at 23° C. Five minutes later, the coffee has cooled to 70° C. At what time will the coffee have cooled down to 40° F?