MATH 136 – Calculus 2

Global Climate Change and Integration

February 3, 2020

Background

This worksheet is inspired by materials developed by Prof. Diana Davis of Swarthmore College. Prof. Davis used them last week in a class meeting of Prof. Roberts' MATH 134 class here at Holy Cross and has given us permission to use and adapt them.

As you should be aware, a major driver of global climate change is emissions of *greenhouse gases* such as CO_2 into the atmosphere from human activities involving burning of fossil fuels for transportation, electric power generation, manufacturing, home heating, and so forth. Global average temperatures are now slightly higher than 1° C above their pre-Industrial Revolution levels. Damaging sea level rise and extreme weather events (such as the recent Australian wildfires) will become a major problem around the world if global average temperature rises more than 1.5° C. Scientists estimate that the *total additional amount* of CO_2 that we can put into the atmosphere without reaching the 1.5° C level is *about* 400 *Gigatons (Gt) of carbon dioxide*. Let's call that our *carbon budget*.

In this discussion, we want to use calculus to analyze the predictions of various model scenarios regarding the *rate of carbon dioxide emissions* (in Gt per year) and the accumulated *total emissions* in each case.

Questions

(A) (A "warmup.") Let a be a constant. Using your knowledge of derivative rules, find a general antiderivative for $f(t) = e^{at}$.

(B) Let t be the number of years after 1/1/2020, so $t = 0$ is close to the present. If current trends continue, we will be emitting CO_2 into the atmosphere at a rate of about

$$f_{\text{current}}(t) = 34 + 1.3 \cdot e^{0.065 \cdot t}$$

Gt per year.

(1) Explain why the *total additional amount of carbon dioxide* emitted between the present time and year $t = b$ in the future will be

$$F(b) - F(0) = \int_0^b f_{\text{current}}(t)\ dt$$

where F is an antiderivative of $f_{\text{current}}(t)$. Plot $y = f_{\text{current}}(t)$ to visualize the value of this integral as an area.

(2) How much carbon dioxide will be emitted between now and 2050?

(3) Estimate when we will exceed the carbon dioxide emissions budget of 400 Gt.

(C) A better plan would be to gradually "wean ourselves off" from fossil fuel burning. If we gradually reduce our emissions, we might come up with a plan to make our rate of emissions at time $t =$ number of years after 2020 go according to this function:

$$f_{\text{better}}(t) = 34 + 1.3 \cdot e^{0.065 \cdot t} - 0.024 \cdot e^{0.25 \cdot t} \text{ Gt/yr.}$$

(1) What is $f_{\text{better}}(30)$? What does this number mean?

(2) What is the total carbon dioxide emitted between 2020 and 2050 under the better plan? (Hint: Compute this with an integral like the one in question B, part 1).

(3) When will the world exceed its carbon budget with this plan?

(D) A *carbon sequestration* plan, in which we actively take carbon dioxide out of the atmosphere (either by planting many trees world-wide, or through other means), might yield an emissions rate like this:

$$f_{\text{sequestration}} = 34 + 1.3 \cdot e^{0.065 \cdot t} - 4 \cdot e^{0.12 \cdot t} \text{ Gt/yr.}$$

(1) Plot $y = f_{\text{sequestration}}(t)$ for $0 \leq t \leq 30$.

(2) Under this plan, when will carbon dioxide in the atmosphere reach its *maximum level*? (Use your graph to estimate.)

(3) Will the world exceed its carbon budget with this plan?

(E) Write a short paragraph explaining the conclusions you derive from these calculations.

Assignment

Group writeups due at the start of class on Wednesday, February 5.