College of the Holy Cross
MATH 135, section 1
Exam 3 – Friday, December 6

Your Name: _____

Instructions: For full credit, you must show *all work* on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values.

Please do not write in the space below

Problem	Points/Poss
I	/ 25
II	/ 30
III	/ 20
IV	/ 25
Total	/100

I. For each of the following functions find the derivative. You do not need to simplify.

A. (5) $f(x) = e^x \sin(5x) + \sec(x)$

$f'(x) =$

B. (5) $g(x) = \dfrac{\cos(x)}{\ln(x)}$ $g'(x) =$

C. (5) $h(x) = \tan^{-1}(x^2 - 4)$ $g'(x) =$

D. (5) Find $\frac{dy}{dx}$ by implicit differentiation if $x^2y^3 + 2y = 3$.

$\frac{dy}{dx} =$

E. (5) Find all the c in the open interval $(0, 4)$ for which the conclusion of the Mean Value Theorem is true for $f(x) = x^3 - 4x^2 + 3x$ on the interval $[a, b] = [0, 4]$.

$c =$

II. A poster is to be printed on a rectangular sheet of paper with 1 inch margins along the sides and 1.5 inch margins at the top and bottom. The rectangular printed region inside the margins must be 500 square inches in area. Find the dimensions of the overall sheet of the smallest possible area meeting these requirements. Let x, y be the width and height *of the printed area*.

A. (5) Draw a picture representing the whole poster and label the sides of length x and y clearly.

B. (10) Express the area of the whole poster in terms of the one variable x–the width of the printed area.

C. (10) Determine a critical point of your function from B. Why is it a minimum?

D. (5) What are the dimensions of the whole poster of minimum area?

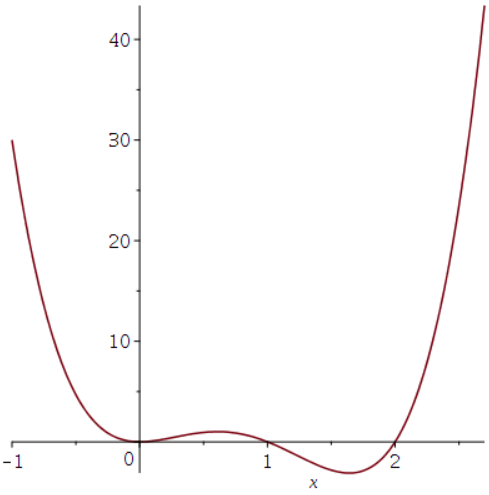

Figure 1: Figure for Problem III.

III. All parts of this question refer to the plot in Figure 1, which shows $y = f'(x)$ for some function f.

A. (10) Recall that the plot shows $y = f'(x)$(!) Approximately where does the graph $y = f(x)$ have inflection points?

Inflection points at $x =$ ____

B. (10) Recall that the plot shows $y = f'(x)$(!) Where are the critical points of f and what types are they (i.e. local maximum, local minimum, neither)?

Answer: ____

IV. All parts of this question refer to the function $f(x) = \dfrac{x}{(2x+1)^2}$, for which the first two derivatives are, in simplified form:

$$\begin{aligned} f'(x) &= \frac{1-2x}{(2x+1)^3} \\ f''(x) &= \frac{8x-8}{(2x+1)^4} \end{aligned}$$

A. (10) Does $y = f(x)$ have any horizontal asymptotes? Use L'Hopital's Rule.

Horizontal asymptote at $y =$ ____

B. (10) What is the maximum value of $f(x)$ on the interval $[0, 3]$?

Maximum value: ____

C. (5) Over which interval(s) is the graph $y = f(x)$ concave up?

Concave up on: ____