College of the Holy Cross, Fall 2019
Math 135, Section 1, Solutions for Midterm 1
Friday, September 27

I. The following table contains values for three different functions: $f(x), g(x), h(x)$.

x	2.5	3.4	4.3	5.2	6.1
$f(x)$	3	9	27	81	243
$g(x)$	12.1	13.2	14.3	15.4	16.5
$h(x)$	5.4	3.6	1.3	5.3	7.9

A) (15) One of these is a linear function. Explain how you can tell which one it is, and give a formula for it.

Solution: The function $g(x)$ is linear since equal increments in x produce equal increments in $g(x)$. Alternatively, the slopes between all pairs of points $(x, g(x))$ from the table are equal. The slope is

$$m = \frac{13.2 - 12.1}{3.4 - 2.5} = \frac{1.1}{.9} \doteq 1.22$$

and the equation of the line can be found by the point-slope form:

$$y - 12.1 \doteq (1.22)(x - 2.5)$$

or $y \doteq 1.22x + 9.05$ (the intercept is closer to 9.04 if more decimal places in the slope are used).

B) (10) One of these functions is *neither linear nor exponential*. Explain which one that is and why.

Solution: Exponential and linear functions are either always increasing or always decreasing. The third function $h(x)$ is neither, so it is the one that is neither linear nor exponential.

II.

A) (10) Simplify using properties of logarithms and exponents. (No credit will be given for only an approximate calculator value.)

$$\log_3\left(\frac{\sqrt{3}}{\sqrt[4]{27}}\right)$$

Solution: By rules for exponents, this the same as

$$\log_3\left(\frac{3^{1/2}}{27^{1/4}}\right) = \log_3\left(\frac{3^{1/2}}{3^{3/4}}\right) = \log_3(3^{-1/4}) = \frac{-1}{4}$$

Simplified form: -1/4

B) (15) The population of a city (in millions) at time t (years) is $P(t) = 3.7e^{0.04t}$. When will the population reach 6.3 million?

Solution: We need to solve the equation

$$3.7e^{0.04t} = 6.3$$

for t. We divide by 3.7, then take natural logs to solve for the t in the exponent:

$$t = \frac{\ln(6.3/3.7)}{.04} \doteq 13.3$$

Time: (a bit more than) 13.3 years

III. Given $f(x) = \dfrac{1}{x^2 - 6x + 8}$ and $g(x) = \tan(x)$, but defined *only for x in the interval* $-3 \le x \le 3$. Answer the following questions.

A) (10) Which x between -3 and 3 must be removed to obtain

Solution: The x-values where $f(x)$ are undefined can be found by either factoring:

$$x^2 - 6x + 8 = (x - 2)(x - 4)$$

or using the quadratic formula. So $f(x)$ is undefined at $x = 2, 4$, but only $x = 2$ is contained in $-3 \le x \le 3$. The function $g(x) = \tan(x) = \sin(x)/\cos(x)$ is undefined at all the odd integer multiples of $\pi/2$ (the zeroes of $\cos(x)$). Of those, only $\pm\pi/2$ are contained in the interval $-3 \le x \le 3$.

the domain of f: $x = 2$ must be removed

the domain of g: $x = \pm\pi/2$ must be removed

B) (10) Using the Limit Laws, determine $\lim_{x \to 1} f(x)g(x)$.

Solution: By the Limit Product Law, and then the Limit Quotient Law and Limit Sum Law for $f(x)$,

$$\begin{aligned}
\lim_{x\to 1} f(x)g(x) &= \lim_{x\to 1} f(x) \cdot \lim_{x\to 1} g(x) \\
&= (\lim_{x\to 1} \frac{1}{x^2 - 6x + 8}) \cdot \lim_{x\to 1} \tan(x) \\
&= \frac{1}{3} \cdot \tan(1) \\
&\doteq .519
\end{aligned}$$

limit: The exact value is $\frac{1}{3}\tan(1) \doteq .519$

IV. (Make sure your calculator is set in radian mode for this problem.)

A) (20) Let $f(x) = \dfrac{\sin(5x)}{8x}$. Compute the values at $x = \pm\pi/10$, $x = \pm\pi/100$, $x = \pm\pi/1000$ accurate to three decimal places and fill in the table below:

x	$-\pi/10$	$-\pi/100$	$-\pi/1000$	$\pi/1000$	$\pi/100$	$\pi/10$
$f(x)$	.398	.622	.625	.625	.622	.398

B) (10) What's your estimate of the value of the limit $\lim_{x \to 0} \dfrac{\sin(5x)}{8x}$ based on this numerical information?

limit: Probably around .625