Holy Cross College, Fall Semester, 2019
MATH 135, Section 01, Final Exam
Saturday, December 21, 8:00 AM

Your Name: _____

Instructions. Clearly mark your answers, and show work on the test itself. Use the back of the preceding page if you need more space for scratch work. *You must show all work for full credit*, but please place answers in the boxes provided where appropriate. The questions that will be included in the Exam III subscore are marked with an asterisk ($*$).

Please do not write in the space below

Problem	Points/Possible
1	/ 20
2	/ 30
3	/ 30
4	/ 30
5	/ 40
6	/ 15
7	/ 20
8	/ 15
Exam III Subscore	/80
Total	/200

Have a peaceful and joyous holiday season!

1. [20 points] One of the functions given in the following table is linear and the other is exponential. Find a formula for the *exponential* one and place it in the appropriate box. In the box for the other one, write "Linear."

x	1	2	3	4	5
$f(x)$	1.2	2.4	4.8	9.6	19.2
$g(x)$	-3.3	-4.4	-5.5	-6.6	-7.7

$f(x) =$

$g(x) =$

2. Let $f(x) = x^3 + 3x + 1$ and $g(x) = \sqrt{x^2 + 3}$.

(a) [10 points] Find the formula for the composition $g(f(x))$

$$g(f(x)) = \boxed{}$$

(b) [10 points] Write $g(x) = h(k(x))$ for two other functions h and k, where $h(x) \neq x$ and $k(x) \neq x$.

$h(x) = \boxed{}$ $\qquad\qquad$ $k(x) =$

$\boxed{}$

(c) [10 points] Find the average rate of change of $g(x)$ per unit change in x on the interval $[0, 2]$.

Average rate of change $= \boxed{}$

3. Compute the following limits [10 points each]. *Any legal method is OK.*

(a) (*) $\lim\limits_{x\to 0}(1+5x)^{1/x}$

Limit =

(b) $\lim\limits_{x\to 3}\dfrac{x^2-6x+9}{x^2-5x+6}$

Limit =

(c) $\lim\limits_{x\to 0}\dfrac{\sin(3x)}{x}$

Limit =

4.

(a) [10 points] State the limit definition of the derivative:

$$f'(x) = \boxed{}$$

(b) [10 points] Use the definition to compute $f'(x)$ for $f(x) = \dfrac{1}{x^2}$.

(c) [10 points] Find the equation of the tangent line to the graph $y = \dfrac{1}{x^2}$ at the point $(2, 1/4)$. Note: you can do this one even if you were not able to complete part b above.

Tangent line: $\boxed{}$

5. Compute the following derivatives using the derivative rules. You need not simplify.

(a) [10 points] $f(z) = z^5 - \dfrac{1}{\sqrt[6]{z}} + e^z$.

$$f'(z) = \boxed{}$$

(b) [10 points] $g(x) = (x^3 + 1)(x^2 - 1)$
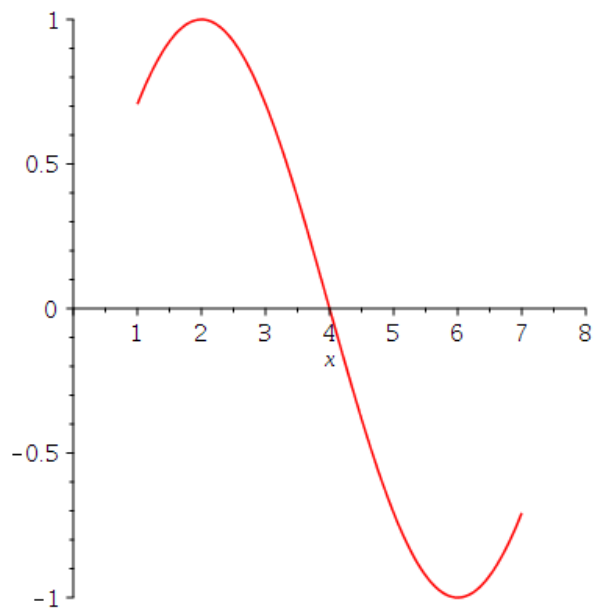
(c) (*) [10 points] $h(t) = \dfrac{\ln(t^2 + 3)}{\sin^{-1}(t)}$

Figure 1: Plot for question 6.

(d) (*) [10 points] Find $y' = \frac{dy}{dx}$ if $3x^3y^2 - 7y^3 + \tan(x) = y$.

6. (*) [5 points each] All parts of this question refer to the function $f(x)$ whose *derivative* $f'(x)$ is plotted in Figure 1 above. Assume the domain of f contains only the interval shown in the plot.

(a) On which interval(s) is $f(x)$ increasing?

Increasing on:

(b) On which interval(s) is $y = f(x)$ concave down?

Concave down on:

(c) How many critical points does f have? Classify them as local maxima, local minima or neither.

Critical points:

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7. (*) [20 points] If the ticket price for the upcoming "Star Wars" movie is set at $10, then 1000 tickets for the first showing will be sold. However, for each $0.25 increase in the ticket price, the number of tickets sold will go down by 10 tickets. Each person who buys a ticket will also purchase $12 of candy, popcorn and drinks at the concession stand. What is the maximum revenue that can be earned by the theater from the ticket and concession sales?
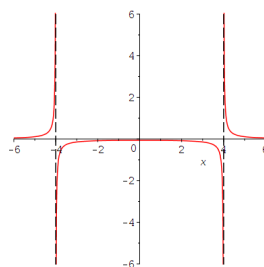
Maximum Revenue:
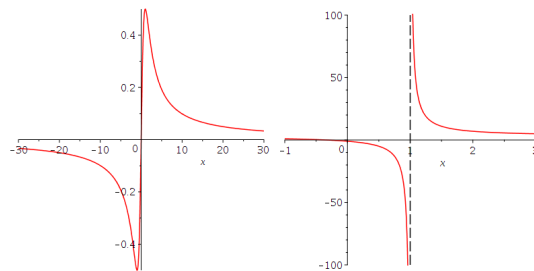
(a) Plot I

(b) Plot II
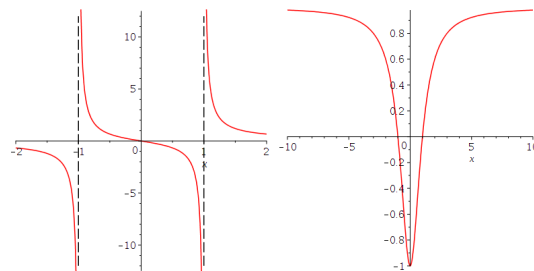
(c) Plot III

(d) Plot IV

(e) Plot V

8. (*) [3 points each]   By considering the location of vertical and horizontal asymptotes, x-and y-axis intercepts, etc. determine which of the following functions matches each graph. Circle the number of the graph showing each of the following functions.

(a) $f(x) = \dfrac{1}{x^2 - 16}$ ⬩⬩⬩⬩⬩   I   II   III   IV   V

(b) $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$ I   II   III   IV   V

(c) $f(x) = \dfrac{x}{x^2 + 1}$ I   II   III   IV   V

(d) $f(x) = \dfrac{x}{x^2 - 1}$ I   II   III   IV   V

(e) $f(x) = \dfrac{3x + 1}{x - 1}$ I   II   III   IV   V