Holy Cross College, Fall Semester, 2019
MATH 135, Section 01, Final Exam
Saturday, December 21, 8:00 AM

Your Name: ______________________________

Instructions. Clearly mark your answers, and show work on the test itself. Use the back of the preceding page if you need more space for scratch work. *You must show all work for full credit*, but please place answers in the boxes provided where appropriate. The questions that will be included in the Exam III subscore are marked with an asterisk (∗).

Please do not write in the space below

Problem	Points/Possible
1	/ 20
2	/ 30
3	/ 30
4	/ 30
5	/ 40
6	/ 15
7	/ 20
8	/ 15
Exam III Subscore	/80
Total	/200

HAPPY HOLIDAYS!

1. [20 points] One of the functions given in the following table is linear and the other is exponential. Find a formula for the *linear* one and place it in the appropriate box. In the box for the other one, write "Exponential."

x	1	2	3	4	5
$f(x)$	1.2	2.4	4.8	9.6	19.2
$g(x)$	-3.3	-4.4	-5.5	-6.6	-7.7

$f(x) =$

$g(x) =$

2. Let $f(x) = x^3 + 3x + 1$ and $g(x) = \sqrt{x^2 + 3}$.

(a) [10 points] Find a formula for the composition $f(g(x))$

$f(g(x)) =$

(b) [10 points] Write $g(x) = h(k(x))$ for two other functions h and k, where $h(x) \neq x$ and $k(x) \neq x$.

$h(x) =$

$k(x) =$

(c) [10 points] Find the average rate of change of $f(x)$ per unit change in x on the interval $[1, 3]$.

Average rate of change =

3. Compute the following limits [10 points each]. *Any legal method is OK.*

(a) $\lim_{x \to 0} \dfrac{\sin(4x)}{x}$

Limit =

(b) $\lim_{x \to 2} \dfrac{x^2 - 8x + 12}{x^2 - 2x}$

Limit =

(c) (*) $\lim_{x \to \infty} \left(1 + \dfrac{3}{x}\right)^x$

Limit =

4.

(a) [10 points] State the limit definition of the derivative:

$f'(x) =$ [ ]

(b) [10 points] *Use the definition* to compute $f'(x)$ for $f(x) = \dfrac{1}{x+1}$.

(c) [10 points] Find the equation of the tangent line to the graph $y = \dfrac{1}{x+1}$ at the point $(2, 1/3)$. Note: you can do this one even if you were not able to complete part b above.

Tangent line: [ ]

5. Compute the following derivatives using the derivative rules. You need not simplify.

(a) [10 points] $f(x) = x^{4/5} - \dfrac{1}{\sqrt[3]{x}} + e^x$.

(b) [10 points] $g(x) = \dfrac{x^2 + 1}{x^4 + 1}$

(c) (*) [10 points] $h(s) = \ln(4s^2 + 2)\tan^{-1}(s)$

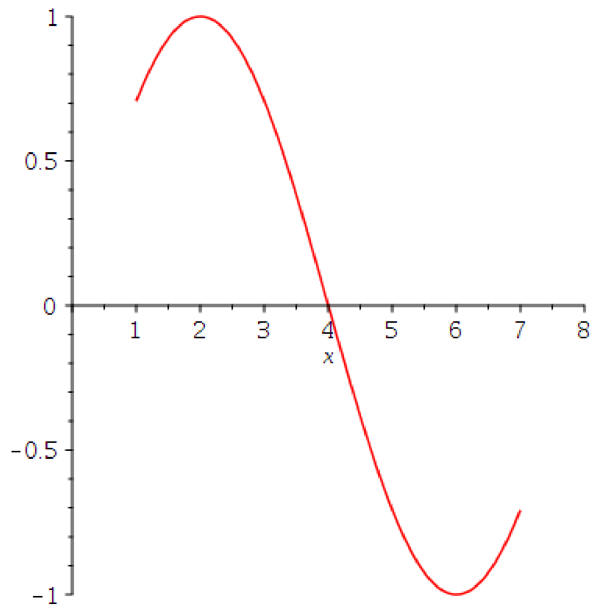

Figure 1: Plot for question 6.

(d) (*) [10 points] Find $y' = \frac{dy}{dx}$ if $4x^2y^3 - 2y^2 + \sin(x) = x^4$.

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6. (*) [5 points each] All parts of this question refer to the function $f(x)$ whose *derivative* $f'(x)$ is plotted in Figure 1 above. Assume the domain of f contains only the interval shown in the plot.

(a) On which interval(s) is $f(x)$ decreasing?

Decreasing on: [ ]

(b) On which interval(s) is $y = f(x)$ concave up?

Concave up on: [ ]

(c) How many inflection points does f have?

Inflection points: [ ]

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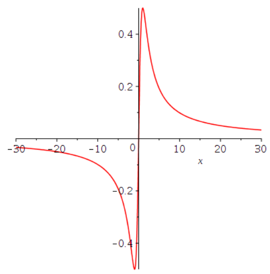
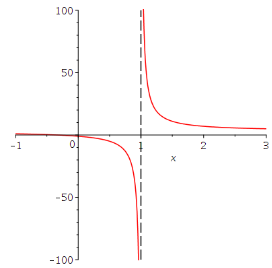
7. (*). [20 points] If the ticket price for the upcoming "Cats" movie is set at \$10, then 1000 tickets for the first showing will be sold. However, for each \$0.25 increase in the ticket price, the number of tickets sold will go down by 10 tickets. Each person who buys a ticket will also purchase \$10 of nachos and drinks at the concession stand. How should the ticket price be set to maximize the revenue earned by the theater from the ticket and concession sales?

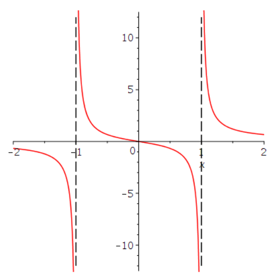
Price: [ ]



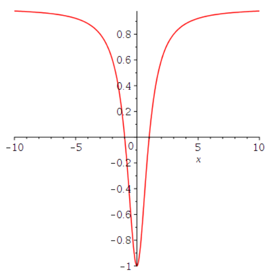
(a) Plot I



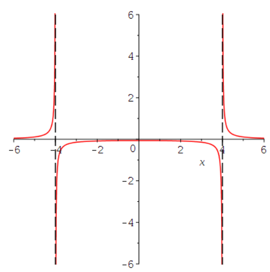
(b) Plot II



(c) Plot III



(d) Plot IV



(e) Plot V

8. (*) [3 points each] By considering the location of vertical and horizontal asymptotes in the plots above, x- and y-axis intercepts, etc. determine which of the following functions matches each graph. Circle the number of the graph showing each of the following functions.

(a) $f(x) = \dfrac{x}{x^2 - 1}$ I II III IV V

(b) $f(x) = \dfrac{x}{x^2 + 1}$ I II III IV V

(c) $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$ I II III IV V

(d) $f(x) = \dfrac{1}{x^2 - 16}$ I II III IV V

(e) $f(x) = \dfrac{3x + 1}{x - 1}$ I II III IV V