**Holy Cross College, Fall Semester, 2016**
**MATH 135, Section 01, Final Exam**
**Friday, December 16, 8:00 AM**

**Your Name**: _____

*Instructions.* Clearly mark your answers, and show work on the test itself. Use the back of the preceding page if you need more space for scratch work. *You must show all work for full credit*, but please place answers in the boxes provided where appropriate.

*Please do not write in the space below*

| Problem | Points/Possible |
|---------|-----------------|
| 1 | / 25 |
| 2 | / 20 |
| 3 | / 25 |
| 4 | / 15 |
| 5 | / 25 |
| 6 | / 20 |
| 7 | / 30 |
| 8 | / 20 |
| 9 | / 20 |
| Exam I Subscore | /60 |
| Total | /200 |

*HAPPY HOLIDAYS!*

(a) Plot I

(b) Plot II



(c) Plot III

(d) Plot IV



(e) Plot V

1. [5 points each]  Circle the number of the graph showing each of the following functions. Note that there is one "extra" graph that does not match any of these functions.

(a) $f(x) = e^{-x} + 2$                               I     II     III     IV     V

(b) $f(x) = x^3 - 4x$                                 I     II     III     IV     V

(c) $f(x) = 2\cos(2\pi x)$                            I     II     III     IV     V

(d) $f(x) = \dfrac{1}{x^2 - 9}$                       I     II     III     IV     V

(e) Give a formula of a function that matches the graph you did not circle.

$$f(x) = \boxed{\phantom{xxxxxxxxxxxxxxxxxxxxxxxx}}$$

2. [20 points] One of the functions given in the following table is linear and the other is exponential. Find a formula for the linear one and place it in the appropriate box. In the box for the other one, write "Exponential."

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $f(x)$ | 1.2 | 0.6 | 0.3 | 0.15 | 0.075 |
| $g(x)$ | $-2.3$ | $-0.6$ | 1.1 | 2.8 | 4.5 |

$f(x) =$ 

$g(x) =$

3.

(a) [15 points] The depth of water in a tank oscillates sinusoidally once every 4 hours according to $d(t) = 2\cos\left(\frac{\pi t}{2}\right) + 4$. Sketch the graph of the depth versus time.

(b) [10 points] Find the average rate of change of the depth on the interval $[1, 1.1]$.

Average rate of change =

4. Compute the following limits [5 points each]. Any legal method is OK.

(a) $\displaystyle\lim_{x\to 2}\frac{x^3+2x}{x-4}$

Limit =

(b) $\displaystyle\lim_{x\to 2}\frac{x^2-7x+10}{x^2-5x+6}$

Limit =

(c) $\displaystyle\lim_{x\to\infty}\frac{5x^2-x+21}{8x^2-9x+1}$

Limit =

5.

(a) [5 points] State the limit definition of the derivative:

$$f'(x) = \boxed{\phantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$

(b) [10 points] Use the definition to compute $f'(x)$ for $f(x) = 3\sqrt{x+2}$.

(c) [10 points] Find the equation of the tangent line to the graph $y = 3\sqrt{x+2}$ at the point $(2, 6)$.

$$\text{Tangent line} = \boxed{\phantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$

6. Compute the following derivatives using the derivative rules. You need not simplify. [5 points each]

(a) $f(t) = t^4 - \dfrac{1}{\sqrt[5]{t}} + e^t$.

$$f'(t) = \boxed{\phantom{xxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$

(b) $g(x) = \dfrac{x^2 - 2}{\cos(x) + 1}$

$$g'(x) = \boxed{\phantom{xxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$

(c) $h(z) = \ln(4z^2 + 2\tan^{-1}(z))$

$$h'(z) = \boxed{\phantom{xxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$

(d) Find $\frac{dy}{dx}$ if $5x^2y^2 - 2y^5 + x = 1$.

$$\frac{dy}{dx} = \boxed{\phantom{xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx}}$$

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7. All parts of this question refer to the functions defined by $f(x) = x^4 + 2ax^2$, where $a$ is any fixed real number.

(a) [10 points] Assuming $a < 0$, find the *critical points* of $f$, and construct a sign diagram for $f'(x)$. Which of your critical points are local maxima and which are local minima?

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(b) [10 points] Repeat part a, but assume now that $a > 0$.

(c) [10 points] How many different *inflection points* does the graph $y = f(x)$ have if $a < 0$?
Explain.

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8. [20 points] The radius and the height of a circular cone increase at a rate of 2 cm/sec.
How fast is the volume of the cone increasing when $r = 10$ and $h = 20$?

Answer:

9. [20 points] A rectangular poster is to have total area 600 square inches, including blank 1 inch wide margins on all four sides of a central printed area. What overall dimensions will maximize the printed area?

Dimensions: