MATH 135 – Calculus 1
Answers for Sample Questions for Exam 1
September 19, 2019

I. Express the set of x satisfying $|2x-5|>1$ as an interval or union of intervals. *Answer:* $|2x-5|>1$ is equivalent to $2x-5>1$ or $2x-5<-1$. The first says $2x>6$, so $x>3$. The second says $2x<4$, so $x<2$. Another way to write this is as the union of the two intervals: $(-\infty,2)\cup(3,\infty)$.

II. The following table contains values for three different functions: $f(x), g(x), h(x)$.

x	0	0.1	0.2	0.3	0.4
$f(x)$	-4.2	-5.9	-7.6	-9.3	-11.0
$g(x)$	10	20	40	80	160
$h(x)$	4	2.3	1.5	2.1	6.1

A) One of these is a linear function. Explain how you can tell which one it is, and give a formula for it.

Answer: $f(x)$ is the linear one, since each change of .1 in x changes $f(x)$ by -1.7. The formula is $f(x)=-17x-4.2$

B) One of these functions is *neither linear nor exponential.* Explain which one that is and why.

Answer: Exponential and linear functions are either increasing for all x or decreasing for all x. That is not true for $h(x)$.

C) Give a possible formula for $g(x)$. (Hint: the values are doubling every time x increases by .1.)

Answer: $g(x)=102^{t/.1}=10(2^{10})^t=10(1024)^t$

III. All parts refer to $f(x)=-3x^2+12x+21$.

A) Where does the graph $y=f(x)$ cross the x-axis?

Answer: By the quadratic formula, when $-3x^2+12x+21=0$, $x=\frac{-12\pm\sqrt{144+252}}{-6}=2\pm\sqrt{11}\doteq -1.317, 5.317$.

B) Where does the graph $y=f(x)$ cross the y-axis.

Answer: This happens when $x=0$, so $y=21$.

D) Sketch the graph $y=-3x^2+12x+21$ for x in $[-4,4]$ and showing correct scales on both the x- and y-axes.

Answer: The graph is a parabola opening down from the vertex $(2,33)$ like this:

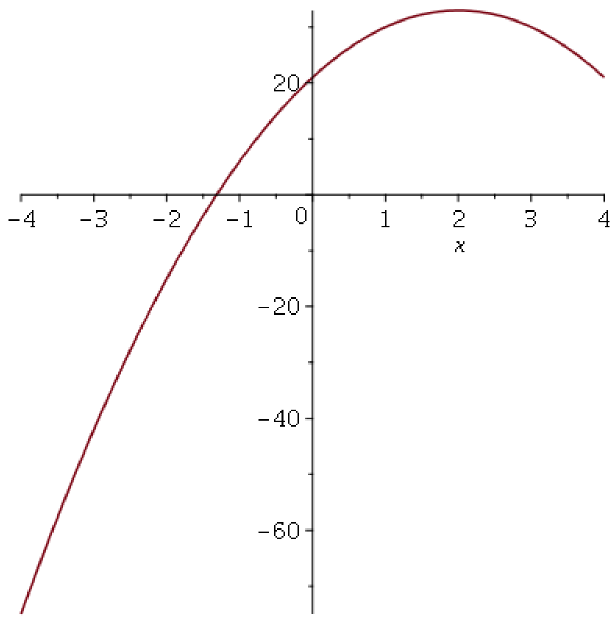

Figure 1: Figure for Question III, part D

IV. You start at $x = 0$ at time $t = 0$ (hours) and drive along the x-axis (x values in miles) at 40 miles an hour for 2 hours. At $t = 2$ you stop for one hour. Then starting at $t = 3$, you retrace your earlier path and return to your starting position at 80 miles per hour.

A) Sketch the graph of your position as a function of time.

Answer: See graph on next page.

B) Give (piecewise) formulas for your function on the appropriate t-intervals.

Answer:

$$x(t) = \begin{cases} 40t & \text{if } 0 \le t \le 2 \\ 80 & \text{if } 2 < t \le 3 \\ 80 - 80(t-3) & \text{if} 3t \le 4. \end{cases}$$

V.

A) Express the domain of the function $f(x) = \frac{x}{x^2-1}$ as a union of intervals.

Answer: It is all $x \neq -1, 1$, so $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

B) The figure for this problem shows the graph $y = \frac{x}{x^2-1}$. Based on this, what can you say about the range of $f(x)$?

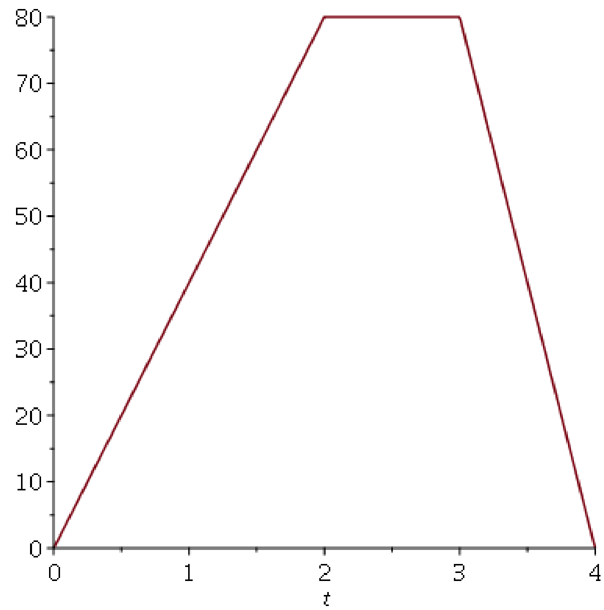

Figure 2: Figure for Question IV,A

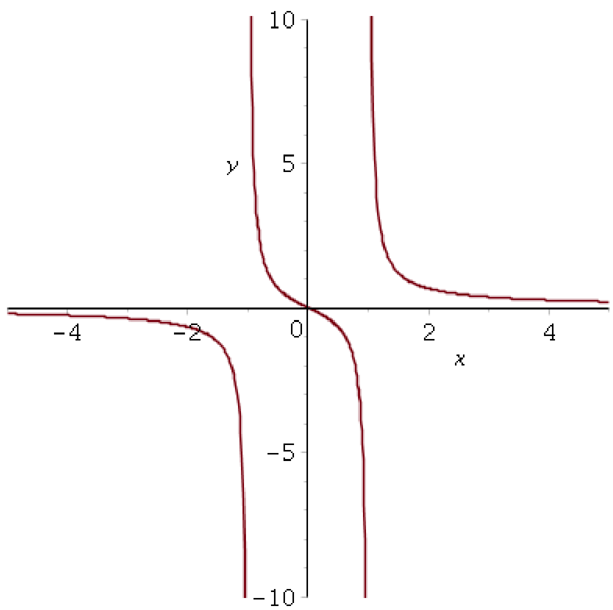

Figure 3: Figure for Question V

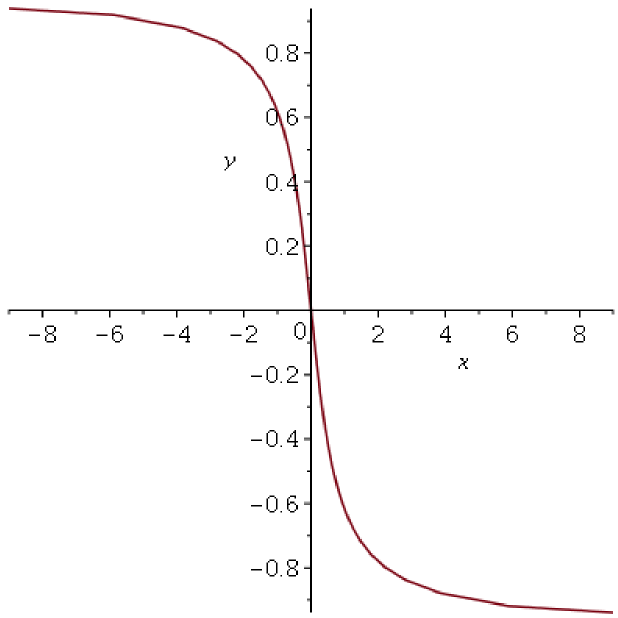

Figure 4: Figure for Question V,D

Answer: Seems to be all real numbers: $\mathbb{R}$, or $(-\infty, \infty)$

C) Explain why $f(x)$ (on its default domain) *fails* to have an inverse function.

Answer: The graph does not pass the horizontal line test, so $f(x)$ is not one-to-one.

D) Give a restricted domain on which $f(x)$ *does* have an inverse function, and sketch the graph of the inverse.

Answer: The interval of x-values $(-1, 1)$ is one such. (The intervals $(1, \infty)$ and $(-\infty, -1)$ would be others.)

VI.

A) What are the *amplitude* and *period* of the sinusoidal function $y = 3\sin\left(\frac{x}{2}\right)$?

Answer: Amplitude = 3, period = 4π.

B) What would change in your answer to B) if the formula was $y = \frac{1}{3}\sin(2x) + 2$?

Answer: The amplitude would change to $\frac{1}{3}$ and the period would change to π.

VII.

A) Simplify: $\log_3(27) + \ln(e^{-3})$.

Answer: 0

B) Solve for x: $2^{x+3} = 3^{x/2}$.

Answer: $x = \frac{6\ln(2)}{\ln(3)-2\ln(2)}$.

C) The population of a city (in millions) at time t (years) is $P(t) = 2.4e^{0.06t}$. What is the population at $t = 0$? When will the population reach 4 million?

Answer: Population at time $t = 0$ is $P(0) = 2.4$ million. The population reaches 4 million at $t = \frac{\ln(4/2.4)}{.06} \doteq 8.5$ years.

D) (Continuation of C) How long will it take for the population to reach double the number at $t = 0$?

Answer: $t \doteq 11.6$ years.

VIII. Let $f(x)$ be the function tabulated below.

x	0	0.1	0.2	0.3	0.4
$f(x)$	−3	−5.9	−7.6	−8	−12.0

A) What is the *average rate of change* of $f(x)$ over the interval $[0.1, 0.2]$?

Answer: The average rate of change is

$$\frac{f(.2) - f(.1)}{.2 - .1} = \frac{-7.6 - (-5.9)}{.1} = -17$$

B) Same question for the interval $[0.2, 0.3]$.

Answer: Compute:

$$\frac{f(.3) - f(.2)}{.3 - .2} = \frac{-8 - (-7.6)}{.1} = -4$$

C) Given the information you have, what is your best estimate for the *instantaneous rate of change* at $t = 0.2$?

Answer: The best estimate would come by averaging the answers from parts A and B, so $\frac{-17+(-4)}{2} = -10.5$.

IX. Investigate

$$\lim_{x\to 0} \frac{2^x - 1}{3^x - 1}$$

numerically by computing the values of $f(x) = \frac{2^x-1}{3^x-1}$ at $x = -.1, -.01, -.001, .001, .01, .1$. What's your estimate of the value of this limit?

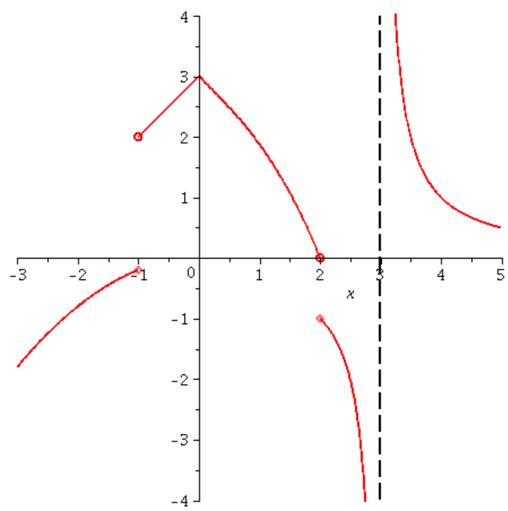

Figure 5: Figure for Question X

Answer: We compute the following values (with a calculator, rounding to 4 decimal places):

x	$-.1$	$-.01$	$-.001$	$.001$	$.01$	$.1$
$f(x)$	.6437	.6322	.6311	.6308	.6297	.6181

From this evidence it seems the limit should be between .6311 and .6308 something like .6310 or .6309. *Note:* With calculus, we will be able to show later that the exact value is $\frac{\ln(2)}{\ln(3)} \doteq .6309$.

X. Consider the function graphed in Figure 5.

A) What is $\lim_{x \to 0} f(x)$?

Answer: $\lim_{x \to 0} f(x) = 3$

B) What are $\lim_{x \to 2^-} f(x)$ and $\lim_{x \to 2^+} f(x)$?

Answer: $\lim_{x \to 2^-} f(x) = 0$ and $\lim_{x \to 2^+} f(x) = -1$

C) What does your answer to part B say about $\lim_{x \to 2} f(x)$?

Answer: It says that $\lim_{x \to 2} f(x)$ does not exist. (The two one sided limits must exist and *be the same* for $\lim_{x \to 2} f(x)$ to exist.)