College of the Holy Cross
MATH 135, section 1 – Calculus 1
Information on Final Exam – December 5, 2016

General Information

- The final examination for this class will be given during the scheduled period 8:00am to 10:30am on Friday, December 16.
- Like the midterms, this one will be given *in our regular classroom*, Swords 359.
- The final will be similar in format to the midterm exams but perhaps twice as long. I expect that if you are well prepared and you work steadily, then you should be able to finish the exam in about 1.75 hours. However, you will have the full 2.5 hour period to work on the exam if you need that much time.
- As on the midterms, you may use a calculator, but no graphing features.
- No cell-phones, computers, or other electronic devices beyond a basic calculator may be used during the exam. Please do not bring them with you; they will be subject to confiscation for the period of the exam if you use them.
- I'm happy to try to find a time for a pre-final review session (probably in the evening). We can discuss this in class on Friday, December 9.
- I will also be available during exam week at the following times, *but ONLY these times,* for "last minute" questions
  - Monday, December 12, 8:00am - 12:00noon
  - Tuesday, December 13, 8:00am - 11:00am
  - Wednesday, December 14, 2:00pm - 4:00pm
  - Thursday, December 15, 2:00pm - 4:00pm

Topics to be Covered

- This will be a *comprehensive* final – it will cover all the topics we have studied this semester, with about 30% each devoted to the material on each of the midterm exams, plus about 10% on the material from Problem Set 9 (especially L'Hopital's Rule and the applied optimization problems).
- See the review sheets for the three midterms for a detailed breakdown of the topics we studied earlier. Those review sheets are now reposted on the course homepage if you need another copy of any of them.

Philosophical Comments and Suggestions on How to Prepare

- The reason we give final exams in almost all mathematics classes is to encourage students to "put whole courses together" in their minds. Also, preparing for the final should help to make the ideas "stick" so you will have the material at your disposal to use in later courses. *This is especially important if you are preparing to continue to MATH 136 – everything you will do there is based on material from this semester's course and it will be difficult or impossible to do well in that course unless you have the material from this one under good control.*

- It may not be necessary to say this, but here goes anyway: *You should take this exam seriously* – it is worth 25% of your course average and it can pull your course grade up *or down* depending on how you do.

- Get started reviewing early and do some work on this *every day* between now and the date of the final. Don't try to "cram" at the end. There's too much stuff that you need to know to approach preparing that way!

- Review videos and your class notes in addition to the text, especially for topics where you lost points on the midterms. There are a lot of worked-out examples and discussions of all of the topics we have covered there.

- Look over the midterm exams with the solutions. Go over your corrected problem sets. If there were questions where you lost a lot of points, be sure you understand why what you did was not correct, and how to solve those questions.

- Be sure you actually do enough practice problems so that you have the facility to solve exam-type questions in a limited amount of time. *Even if you have saved solutions for practice problems from the midterms*, it is going to be much more beneficial to do practice problems starting "from scratch" rather than just reading old solutions. Remember, the goal of the course is to get you to be able to develop solutions to these problems yourselves, not just to understand solutions that someone else (that includes you, one or more months ago!) has written down. Another analogy – as most of you know from your study of languages, it's much easier to understand another language passively than it is to actually use a language actively yourself (for instance, to form your own complete, grammatically correct sentences). The goal of this course is to make you reasonably proficient "calculus speakers" and there's no substitute for active practice on those skills.

- The following is (a slightly edited version of) the final I gave in Calculus 1 in fall 2013. It's a good guide for what our exam will look like.

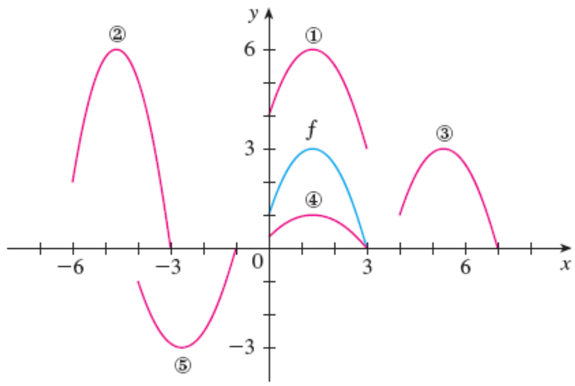

Figure 1: Figure for problem I

I. The graph $y = f(x)$ is given in blue (more like cyan). Match each equation with one of the numbered pink (actually, magenta) graphs.

A) $y = f(x - 4)$ is plot number: ________

B) $y = f(x) + 3$ is plot number: ________

C) $y = \frac{1}{3}f(x)$ is plot number: ________

D) $y = -f(x + 4)$ is plot number: ________

E) $y = 2f(x + 6)$ is plot number: ________

II. A cup of hot chocolate is set out on a counter at $t = 0$. The temperature of the chocolate t minutes later is $C(t) = 70 + 80e^{-t/3}$ (in degrees F).

A) What is the temperature of the chocolate at $t = 0$?

B) What is the rate of change of the temperature at $t = 10$ minutes?

C) How long does it take for the temperature to reach $100°F$?

III. Compute the following limits. Any legal method is OK.

(A) $\displaystyle\lim_{x \to 3} \frac{x^2 + x - 12}{x^2 - 5x + 6}$.

(B) $\displaystyle\lim_{x \to 1^-} \frac{|x - 1|}{x^2 - 1}$.

(C) $\displaystyle\lim_{x \to 0} \frac{\tan(x)}{x}$.

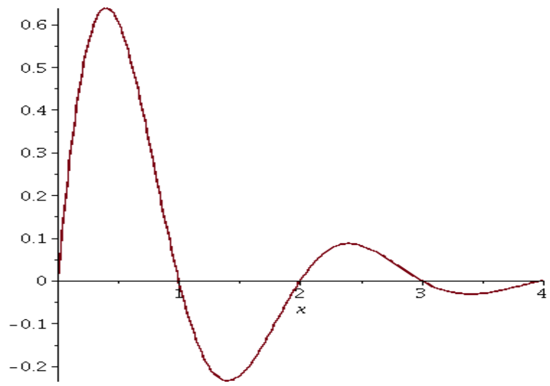

Figure 2: Figure for problem V.

(D) $\lim_{x\to\infty} \dfrac{x^2+3x}{e^x}$.

IV.

A) *Using the limit definition*, and showing all necessary steps to justify your answer, compute $f'(x)$ for $f(x) = 5x^2 - x + 3$.

Using appropriate derivative rules, compute the derivatives of the following functions. You do not need to simplify your answers.

B) $g(x) = 4x^3 + \sqrt{x} + \dfrac{2}{\sqrt[4]{x}} + e^2$

C) $h(x) = \dfrac{\sin(x) + x}{\sec(x)}$

D) $i(x) = (x^2 + 4e^x)\ln(x^3 + 3)$

E) $j(x) = \tan^{-1}(12x + 2)$

F) Find $\frac{dy}{dx}$ by implicit differentiation if

$$xy^3 + 3x^2y^4 + y = 1$$

and find the equation of the tangent line to this curve at $(x, y) = (1, -1)$.

V. The graph in Figure 2 shows the *derivative* $f'(x)$ for some function $f(x)$ defined on $0 \le x \le 4$. Note: This *is not* $y = f(x)$, it is $y = f'(x)$. Using the graph, *estimate*

A) The interval(s) on which $f(x)$ is increasing.

B) The critical points of $f(x)$ *in the open interval* $(0, 4)$. Say what the behavior of $f(x)$ is at each critical number (local max, local min, neither).

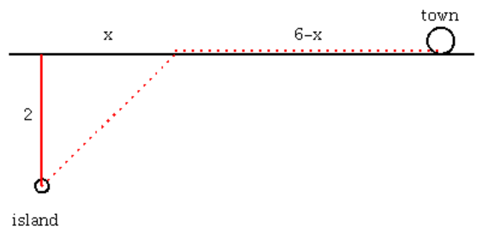

Figure 3: Figure for problem VI.

C) The interval(s) on which $y = f(x)$ is concave down.

VI. A town wants to build a pipeline from a water station on a small island 2 miles from the shore of its water reservoir to the town. One possible route is shown dotted in red. The town is 6 miles along the shore from the point nearest the island. It costs $3 million per mile to lay pipe under the water and $2 million per mile to lay pipe along the shoreline.

A) Give the cost $C(x)$ of the pipeline as a function of the location x as shown.

B) Where along the shoreline should the pipeline hit land to minimize the costs of construction? Say how you know this gives the minimum cost.

VII. A block of dry ice (solid CO_2) is evaporating and losing volume at the rate of 10 cm^3/min. It has the shape of a cube at all times. How fast are the edges of cube shrinking when the block has volume 216 cm^3?

VIII. True or false: The graph obtained by stretching $y = e^{-x}$ vertically by a factor of 2 can *also* be obtained from $y = e^{-x}$ by a horizontal shift. Explain your answer.