MATH 135 – Calculus 1
Sample Questions for Exam 2
October 22, 2019

1. An object moves along a straight line path with position given by $x(t) = 4t^2 + t - 7$, (t in seconds, x in feet).

   (a) What is the average velocity of the object over the interval $[0, 5]$ of t-values?

   (b) Fill in the following table with average velocities computed over the indicated intervals. Using this information, estimate the *instantaneous velocity* at $t = 2$.

interval	$[2, 2.5]$	$[2, 2.05]$	$[2, 2.005]$	$[2, 2.0005]$
ave.vel.				

   (c) Construct a similar table for intervals *ending* at $t = 2$ and repeat the calculations in the previous part. If you estimate the instantaneous velocity at $t = 2$ using this new information, does your result agree with what you did before (it should!)

2. (a) What is the slope of the secant line to the graph $y = x^3 + 1$ through the points with $x = 1$ and $x = 2$?

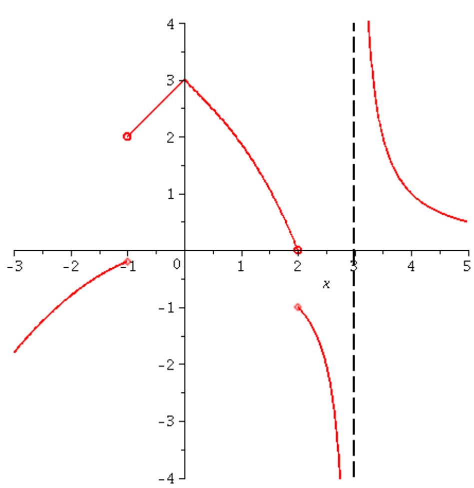
   (b) What is the slope of the secant line to the graph $y = x^3 + 1$ through the points with $x = 1$ and $x = 1 + h$ for a general h?

   (c) The slope of the tangent line to $y = x^3 + 1$ at $x = 1$ would be obtained from what limit?

   (d) Estimate the limit in the previous part numerically (as in the first question).

   (e) Evaluate the limit exactly using our algebraic techniques.

3. The graph of a function f is shown below with several points marked. Find all the marked points at which the following are true, and give explanations for your answers.



(a) f has an infinite discontinuity
(b) f has a jump discontinuity. Find the one-sided limits at each such point.
(c) f has a removable discontinuity
(d) f is continuous

4. Compute the indicated limits. Show all work for full credit.

(a) $\displaystyle\lim_{x\to 1} \frac{3x^2-5x-2}{x^2-4x+4}$

(b) $\displaystyle\lim_{x\to 2} \frac{3x^2-5x-2}{x^2-4x+4}$

(c) $\displaystyle\lim_{x\to \infty} \frac{3x^2-5x-2}{x^2-4x+4}$ What does your answer say about horizontal asymptotes to the graph $y = \dfrac{3x^2-5x-2}{x^2-4x+4}$?

(d) $\displaystyle\lim_{x\to 2} \frac{\frac{1}{x^2}-\frac{1}{4}}{x-2}$

(e) $\displaystyle\lim_{t\to 0} \frac{\sin(6t)}{\sin(7t)}$

(f) $\displaystyle\lim_{h\to 0} \frac{\sqrt{h+9}-\sqrt{9}}{h}$

5. Suppose you know each of the following conditions. What can you say about $\lim_{x\to c} f(x)$ for the indicated c? Why?

(a) $x^2 + x \le f(x) \le x^3 + 3$ for all real x, at $c = 0$.
(b) $-x^2 + 2x \le f(x) \le x^4 - 4x^3 + 6x^2 - 4x + 2$ for all real x, at $c = 1$
(c) $f(x) = x\sin\left(\frac{1}{x}\right)$ for all real $x \ne 0$, at $c = 0$

6. (a) What is the definition of the derivative of a function f at $x = a$ in its domain?
(b) Using the definition (not the shortcut rules), find $f'(x)$ for $f(x) = 3x^3 - 2x^2 + 1$ at a general $x = a$. Check your answer using the shortcut rules.
(c) Using the definition (not the shortcut rules), find $f'(x)$ for $f(x) = x^{1/2}$ at a general $x = a > 0$. Check your answer using the shortcut rules.
(d) Using the definition (not the shortcut rules), find $f'(x)$ for $f(x) = \frac{1}{x^2}$ at a general $x = a \ne 0$. Check your answer using the shortcut rules.

7. (a) State and prove the product rule for derivatives.

   (b) Use the product rule to find $f'(x)$ for $f(x) = (4x^3 - 12x^2 + 1)e^x$

   (c) Find $f'(x)$ for

$$f(x) = \frac{x^2 - 4x + 1}{x^3 + 2}$$

   (d) Find $f'(x)$ two ways: One, using the quotient rule, one *without* using the quotient rule. Verify that the result is the same in both cases. Which is easier?

$$f(x) = \frac{x^6 - 3x^3 + x}{x^{1/2}}$$