MATH 133, section 1 – Calculus With Fundamentals 1
Exam 3 Practice Problems
November 3, 2017

I. Do not use the differentiation rules from Chapter 3 in this question.

A) State the limit definition of the derivative $f'(x)$.
B) Use the definition to compute the derivative function of $f(x) = \frac{1}{3x}$.
C) Find the equation of the line tangent to the graph $y = \frac{1}{3x}$ at $x = 2$.

II. Use the sum, product, quotient, and/or chain rules to compute the following derivatives. You may use any correct method, but must show work for full credit.

A)
$$\frac{d}{dx}\left(5x\sqrt{x} - \frac{2}{x^3} + 11x - 4\right)$$

B)
$$\frac{d}{dt}\left(\frac{t^2 e^{3t}}{t^4 + 1}\right)$$

C)
$$\frac{d}{dz}\frac{z^2 - 2z + 4}{z^2 + 1}$$

D)
$$\frac{d}{dx}\left(\frac{\pi^2 + \tan(e^\pi) - 2x^e}{4}\right)$$

E)
$$\frac{d}{dx}\left(\sin(x)\left(x^7 - \frac{4}{\sqrt{x}}\right)\right)$$

F) Find y' (note this is just another way of asking the same question!)
$$y = (e^{2x} + 2)^3$$

G) Find y'
$$y = \frac{x + 1}{3x^4 - 1}$$

H) Find y'
$$y = \frac{\sin(x)}{1 + \cos(x)} + x^2 \cos(x^3 + 3)$$

III. The total cost (in \$) of repaying a car loan at interest rate of $r\%$ per year is $C = f(r)$.

A) What is the meaning of the statement $f(7) = 20000$?

B) What is the meaning of the statement $f'(7) = 3000$? What are the units of $f'(7)$?

IV. The quantity of a reagent present in a chemical reaction is given by $Q(t) = t^3 - 3t^2 + t + 30$ grams at time t seconds for all $t \geq 0$. (Note: For a question like this, I could also give you the plot of the function and ask questions like those below. In this case you need to start from the formula and compute $Q'(t)$; if you were given the graph, you need to make the connection between slopes of tangent lines and signs of $Q'(t)$ visually.)

(a) Over which intervals with $t \geq 0$ is the amount increasing? (i.e. $Q'(t) > 0$) decreasing (i.e. $Q'(t) < 0$)?
(b) Over which intervals is the rate of change of Q increasing? decreasing? decreasing?

V. A spherical balloon is being inflated at 20 cubic inches per minute. When the radius is 6 inches, at what rate is the radius of the balloon increasing? At what rate is the surface area increasing? (The volume of a sphere of radius r is $V = \frac{4\pi r^3}{3}$ and the surface area is $A = 4\pi r^2$.)

VI. Review problems 2 and 3 from Quiz 5.