College of the Holy Cross, Fall 2016
MATH 136, Section 02, Final Exam
Tuesday, December 13, 11:30am

Your Name: ______________________________

Instructions: For full credit, you must show *all work necessary to justify your answers* on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values. There are 200 regular points and 10 Extra Credit points.

Please do not write in the space below

Problem	Points/Possible
I	/ 20
II	/ 20
III	/ 60
IV	/ 20
V	/ 30
VI	/ 30
VII	/ 20
Total	/200
Extra Credit	/ 10
Exam I Subscore	/ 60

Have a peaceful and joyous holiday season!

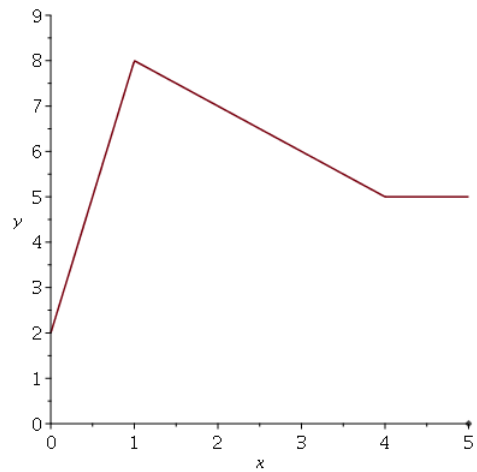

Figure 1: Figure for problem I, parts B and C.

I.

(A) (5) Compute the derivative of the function $g(x) = \displaystyle\int_0^{3x} \frac{\sin(t)}{t}\, dt$.

$$g'(x) = \boxed{}$$

(B) (10) Let $f(x) = \begin{cases} 6x+2 & \text{if } 0 \le x \le 1 \\ -x+9 & \text{if } 1 < x \le 4 \\ 5 & \text{if } 4 < x \le 5 \end{cases}$. which is plotted in the graph above. Let $F(x) = \int_0^x f(t)\, dt$, where $f(t)$ is the function above. Complete the following table of values for $F(x)$:

x	0	1	2	3	4	5
$F(x)$	0					

(C) (5) On which interval(s) contained in $(0, 5)$ is the graph $y = F(x)$ from part B concave down?

Concave down on: $\boxed{}$

II.

(A) (5) Use a left-hand Riemann sum with $n = 4$ to approximate $\int_0^1 e^{-x^2/2}\,dx$.

Answer

(B) (5) Use a midpoint Riemann sum with $n = 4$ to approximate $\int_0^1 e^{-x^2/2}\,dx$.

Answer

(C) (10) Check the appropriate boxes and fill in the blank:

The left-hand sum is an overestimate □/underestimate □,

because: ______________________________________

The midpoint approximation is a overestimate □/underestimate □, because

$e^{-x^2/2}$ is concave up □/concave down □ on $[0, 1]$.

III. Compute the following integrals; full credit only for showing all work leading to the final answer.

(A) (5) $\displaystyle\int \frac{x^{2/3} - x^4 + \sqrt{x}}{x^{2/3}}\,dx$

Answer $\boxed{}$

(B) (5) $\displaystyle\int x\cos(x^2)\,dx$

Answer $\boxed{}$

(C) (10) $\displaystyle\int \frac{\sec^2(5x)\,dx}{(\tan(5x)+7)^5}$

Answer

(D) (10) $\displaystyle\int_1^{e^2} x^3 \ln(x)\,dx.$

Answer

(E) (15) $\displaystyle\int \frac{x^2}{\sqrt{16 - x^2}}\,dx$

Answer

(F) (15) $\displaystyle\int \frac{x+1}{(x-3)(x^2+1)}\,dx$

Answer

IV. For each of the following improper integrals, set up and evaluate the appropriate limits to determine whether the integral converges. If so, find its value; if not, say "does not converge."

(A) (10) $\displaystyle\int_1^3 \frac{1}{\sqrt[3]{x-1}}\,dx.$

Answer

(B) (10) $\displaystyle\int_0^\infty \frac{1}{x^2+4x+5}\,dx.$ (Hint: There are two ways to do this. You can compute the integral if you complete the square. Or you can use $\frac{1}{x^2+4x+5} < \frac{1}{x^2}$ for all $x > 0$.)

Answer

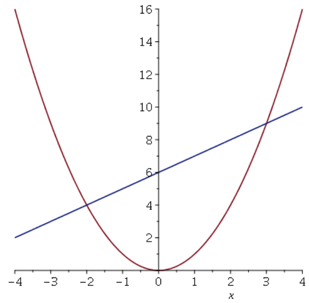

Figure 2: Figure for problem V.

V. A region R in the plane is bounded by the graphs $y = x^2$, $y = x + 6$. See Figure 2.

(A) (10) Compute the area of the region R.

Area = $\boxed{}$

(B) (10) Compute the volume of the solid obtained by rotating R about the x-axis.

Volume = $\boxed{}$

(C) (10) Set up the integral(s) to compute the volume of the solid obtained by rotating R about the line $y = 12$. *You do not need to compute the value.*

Volume integral = $\boxed{}$

VI.

(A) (10) A drug is administered to a patient intravenously at a constant rate of 10mg per hour. The patient's body breaks down the drug and removes it from the bloodstream at a rate proportional to the amount present. Write a differential equation for the function $y(t)$ = amount of the drug present (in mg) in the bloodstream at time t (in hours) that describes this situation. You *do not need to solve* the equation.

Answer

(B) (10) Find the general solution of the differential equation $\frac{dy}{dx} = xy\sqrt{x^2+1}$.

General solution =

(C) (10) Let $y(t)$ represent the population of a colony of tree frogs that is undergoing logistic growth following the differential equation $\frac{dy}{dt} = (.1)y(1 - \frac{y}{100})$, t in years. If the initial population is $y(0) = 10$, how long does it take for the population to reach 45?

Time to reach 45 = $\boxed{}$

VII. For full credit, you must justify your answer completely by showing how the indicated test applies and leads to your stated conclusion.

(A) (5) Use the Integral Test to determine if $\sum_{n=1}^{\infty} \frac{n}{e^{2n}}$ converges.

Answer $\boxed{}$

(B) (15) Using the Ratio Test and testing convergence at the endpoints, determine the interval of convergence for the power series

$$\sum_{n=1}^{\infty} \frac{x^n}{n3^n}.$$

Interval of convergence $\boxed{}$

Extra Credit. (10) In economics, the *multiplier effect* refers to the fact that when there is an injection of money to consumers in an economy, the consumers spend a certain proportion of it. That amount recirculates through the enconomy and adds additional income, which comes back to the consumers and they spend the same percentage. The process repeats *indefinitely* circulating additional money through the economy. Suppose that in order to stimulate the economy, the government cuts taxes by \$ 50 billion, thereby injecting that much money back to consumers. If consumers save 10% of the money they get and spend the other 90%, what is the total additional spending circulated through the economy by the tax cut?