College of the Holy Cross, Fall 2016
Math 136, section 2 – Midterm Exam 2 Retest, version 2
Thursday, November 10

Your Name: ______________________________

Instructions: For full credit, you must show *all work* on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values. I will be liberal with partial credit, so even if you don't see how to complete a problem, do as much as you can and be sure to write what you know.

Please do not write in the space below

Problem	Points/Poss
I	/ 60
II	/ 30
III	/ 10
Total	/100

I.

A. (15) Integrate by parts: $\int \sin^{-1}(x)\ dx$. (Note there's really only one choice for u and hence dv.)

Answer:

B. (15) Integrate using appropriate trigonometric identities: $\int \sin^6(2x)\cos^3(2x)\ dx$.

Answer:

C. (20) Integrate with a trigonometric substitution: $\displaystyle\int \frac{1}{\sqrt{16+x^2}}\,dx$. (Partial credit points will be given as follows: 5 points for the substitution equation $x = \ldots$, and the computation of dx, 5 points for the conversion to the trigonometric integral, 5 points for the integration of the trigonometric integral, 5 points for conversion back to the equivalent function of x.)

Answer: $\boxed{}$

D. (5) Derive the reduction formula (assume $m \geq 2$ – this one is *not* done by parts):

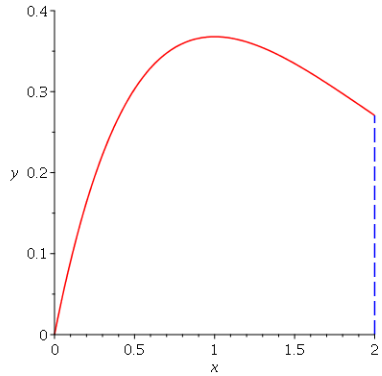
$$\int \tan^m(x)\ dx = \frac{\tan^{m-1}(x)}{m-1} - \int \tan^{m-2}(x)\ dx.$$

Answer:

E. (5) Use the formula from part D repeatedly to integrate $\int \tan^5(x)\ dx$

Answer:

II. All parts of this question refer to the region R bounded by $y = xe^{-x}$ (in red), the x-axis, $x = 0$ and $x = 2$.



A. (10) Set up and compute integral(s) to find the area of R.

Area =

B. (10) The region R is rotated about the x-axis to generate a solid. Set up and compute an integral to find its volume.

Volume = $\boxed{}$

C. (10) A solid has the portion of the region R as base. The cross-sections by planes perpendicular to the x-axis are semicircles with diameter extending from the x-axis up to the point on the graph $y = xe^{-x}$. *Set up* an integral to find the volume. You do not need to compute this one, but if you do correctly, I will give 10 points Extra Credit.

Volume = $\boxed{}$

III. (10) The area of a region in the plane is equally distributed about a point called its *centroid*. For a region R bounded by $y = f(x) \geq 0$, the x-axis, $x = a$ and $x = b$, take it as known that the x-coordinate of the centroid is computed by this ratio of two integrals:

$$\overline{x} = \frac{\int_a^b x f(x)\ dx}{\int_a^b f(x)\ dx}.$$

Pappus's Theorem says: *The volume of the solid generated by rotating R about any line is equal to the product of the area of R and the distance traveled by the centroid as R moves around the axis of the rotation.* Explain how the formula for the x-coordinate of the centroid and Pappus's Theorem show that the volume of the solid obtained by rotating R about the y-axis can be computed by the integral

$$V = \int_a^b 2\pi x f(x)\ dx$$

(giving an alternative to converting to integrals in terms of y).[1]

[1]Pappus's Theorem is a famous result of the ancient Greek mathematician Pappus of Alexandria who lived ca. 290 - 350 CE. He proved it without integral calculus, which had not yet been invented! The formula you are asked to prove is an alternative method for volumes often called the *cylindrical shell* method.