College of the Holy Cross, Fall 2016 Math 136 – Makeup Midterm Exam 3 December 5

Your Name: _____

Instructions: For full credit, you must show *all work* on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values.

Please do not write in the space below

Problem	Points/Poss
Ι	/ 20
II	/ 30
III	/ 20
IV	/ 10
V	/ 20
Total	/100

I. [20 points] Integrate with the partial fraction method: $\int \frac{2x+3}{x^3+4x} dx$

Answer:

II. For each integral, say why the integral is *improper*, then set up and evaluate the appropriate limits to determine whether the integral converges. If so, find its value; if not, say "does not converge." (Full credit will be given only for the correct limit calculation.)

A. [15 points]
$$\int_0^1 \frac{1}{\sqrt{1-x^2}} \, dx.$$

Why improper:

Answer:

B. [15 points] $\int_4^{\infty} \frac{1}{(x-2)(x-3)} dx$ Why improper:

Answer:

III. Both parts of this problem deal with the differential equation $\frac{dy}{dx} + 2y = e^{-3x}$.

A. [15 points] Find the general solution of the equation.

B. [5 points] Find the particular solution y(x) satisfying the initial condition y(0) = 4.

Particular solution:

IV. [10 points] A population y satisfies the logistic growth equation

$$\frac{dy}{dt} = (.2)y\left(1 - \frac{y}{50}\right)$$

and the initial condition y(0) = 1. What is the population level when the population is growing the fastest?

Population level for fastest growth:



A. [10 points] A function y = f(x) is plotted below.



Check the appropriate boxes for statements about $\int_0^3 f(x) \ dx$:

The trapezoidal rule approximation is an overestimate \Box /underestimate \Box .

The midpoint rule approximation is an overestimate \Box /underestimate \Box .

B. [10 points] Suppose you know that the function f(x) plotted here is g''(x) for some other function g(x). How big would you need to take N to get a trapezoidal rule approximation to $\int_0^3 g(x) dx$ with error $< 10^{-3}$?

 $N \geq$