

College of the Holy Cross, Fall 2016
Math 136, section 2, Midterm Exam 1
Friday, September 23

Your Name: _____

Instructions: For full credit, you must show *all work* on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values. The following power sum formulae may be useful:

$$\sum_{j=1}^N j = \frac{N^2}{2} + \frac{N}{2} \tag{1}$$

$$\sum_{j=1}^N j^2 = \frac{N^3}{3} + \frac{N^2}{2} + \frac{N}{6} \tag{2}$$

$$\sum_{j=1}^N j^3 = \frac{N^4}{4} + \frac{N^3}{2} + \frac{N^2}{4}. \tag{3}$$

Please do not write in the space below

Problem	Points/Poss
I	/ 20
II	/ 30
III	/ 20
IV	/ 30
Total	/100

I.

- A. (10) The following limit of a sum would equal the definite integral $\int_a^b f(x) dx$ for some function $f(x)$ on some interval $[a, b]$. What function and what interval?

$$\lim_{N \rightarrow \infty} \sum_{j=1}^N \left(37 - 3 \left(1 + \frac{2j}{N} \right)^2 \right) \cdot \frac{2}{N}.$$

$f(x) =$

$[a, b] =$

- B. (5) For any given N (i.e. without taking the limit), is the sum in part A greater than or less than the integral? Explain how you can tell.

- C. (5) Would you expect the Midpoint Riemann sum or the sum in part A (i.e. before taking the limit as $N \rightarrow \infty$) to be closer to the value of your integral? Explain.

II. All parts of this problem refer to $f(x) = x^2 + 1$ on the interval $[a, b] = [0, 2]$.

A. (10) Evaluate the R_4 Riemann sum for f on this interval.

$R_4 =$

B. (15) Find a formula for R_N not involving a summation; use it to compute $\int_0^2 x^2 + 1 \, dx$.

$R_N =$

Integral =

C. (5) Use Part II of the Fundamental Theorem of Calculus to check your answer from part B.

III. All parts of this problem refer to $f(x) = (x - 2)^2(x - 4)$ and

$$A(x) = \int_0^x f(t) dt$$

for this f .

- A. (10) Where does $A(x)$ have critical points? Is each of them a local maximum, an local minimum, or neither?

Critical points at $x =$

local max/min/neither

- B. (5) Is $A(4)$ a positive or negative number? How can you tell? (It's not necessary to compute the value to tell – why not?)

Answer:

- C. (5) If $B(x) = \int_1^x f(t) dt$, how are $A(x)$ and $B(x)$ related to each other?

Answer:

IV.

A. (10) Integrate with a suitable u -substitution: $\int_0^1 (4x^3 + 1)^{3/5} x^2 dx$.

Integral =

B. (10) Integrate with a suitable u -substitution: $\int \frac{x \sin(3x^2)}{\cos(3x^2) + 1} dx$.

Integral =

C. (10) Integrate with any applicable method we have discussed: $\int_0^1 \frac{x}{\sqrt{x^2 + 1}} dx$

Integral =