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Math 136
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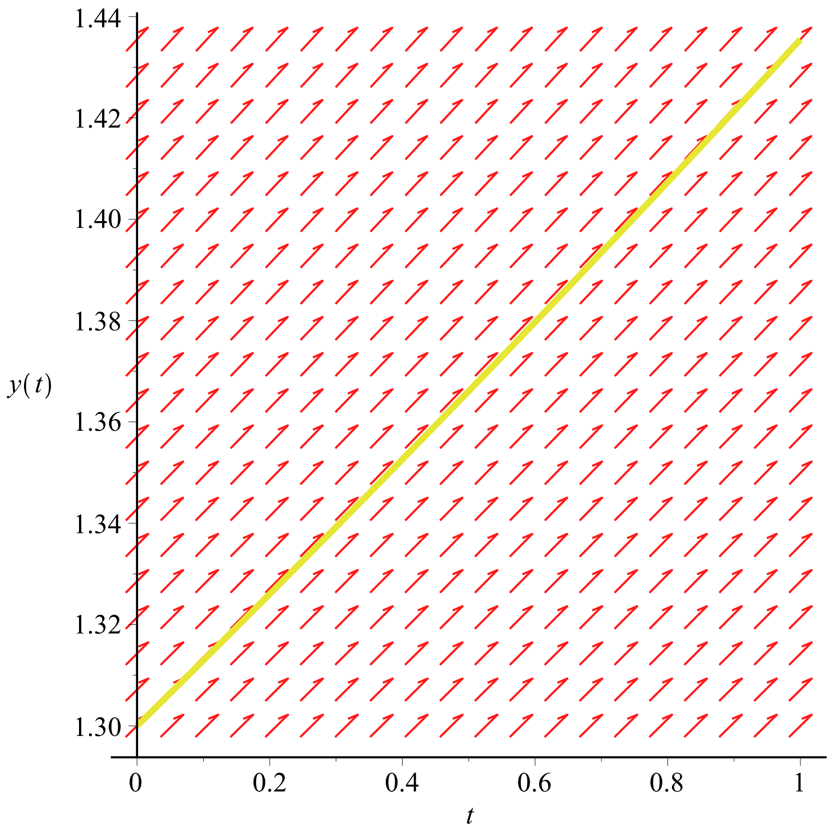
$with(DEtools):$

A)1)

$DEplot\left(diff(y(t), t) = .1 \cdot y(t) \cdot \left(1 - \frac{y(t)}{160}\right), y(t), t = 0..1, [[y(0) = 1.3]]\right)$



2)

$Qsol := t \rightarrow \frac{160}{\left(1 + \left(\frac{160}{1.3} - 1\right) \cdot \exp(-.1 \cdot t)\right)}$

$$t \rightarrow \frac{160}{1 + \left(\frac{160}{1.3} - 1\right) e^{(-1) \cdot 0.1\, t}} \quad \textbf{(1)}$$

$Qsol(0)$

$$1.300000000 \quad \textbf{(2)}$$

$Qsol(10)$

$$3.485110608 \quad (3)$$

$Qsol(20)$

$$9.131734710 \quad (4)$$

$Qsol(30)$

$$22.60572803 \quad (5)$$

$Qsol(40)$

$$49.44502838 \quad (6)$$

$Qsol(50)$

$$87.78919086 \quad (7)$$

$Qsol(60)$

$$122.8314078 \quad (8)$$

$Qsol(70)$

$$143.9729661 \quad (9)$$

$Qsol(80)$

$$153.7054181 \quad (10)$$

$Qsol(90)$

$$157.6252978 \quad (11)$$

$Qsol(100)$

$$159.1181222 \quad (12)$$

$Qsol(110)$

$$159.6744411 \quad (13)$$

$Qsol(120)$

$$159.8800792 \quad (14)$$

$Qsol(130)$

$$159.9558627 \quad (15)$$

$Qsol(140)$

$$159.9837600 \quad (16)$$

$Qsol(150)$

$$159.9940252 \quad (17)$$

3)
$0.99 \cdot 160$

$$158.40 \quad (18)$$

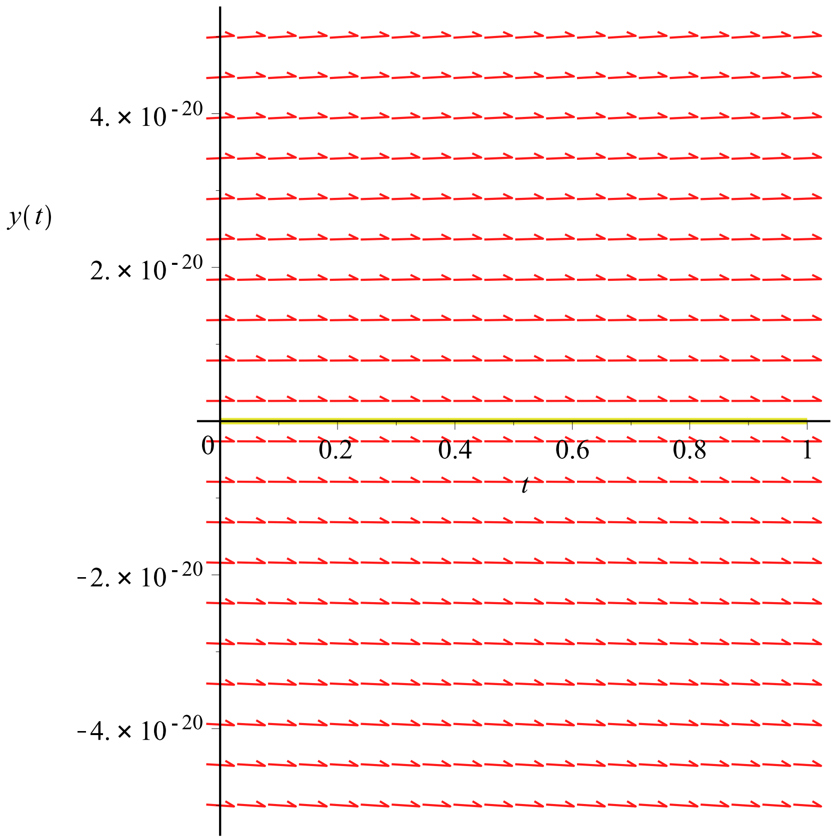
$Qsol(94)$

$$158.4003624 \quad (19)$$

Recovery time is 94 years for this forest.

B)
1) The appropriate intitial condition is Q(0)=0 because all 160Mg of hardwood is mature and therefore one can cut it all down and since the carrying capcaity of max. of Q(t)=160, if you cut it all down there'll be none left at the start so...

$$DEplot\left(diff(y(t), t) = .1 \cdot y(t) \cdot \left(1 - \frac{y(t)}{160}\right), y(t), t = 0..1, [[y(0) = 0]]\right)$$

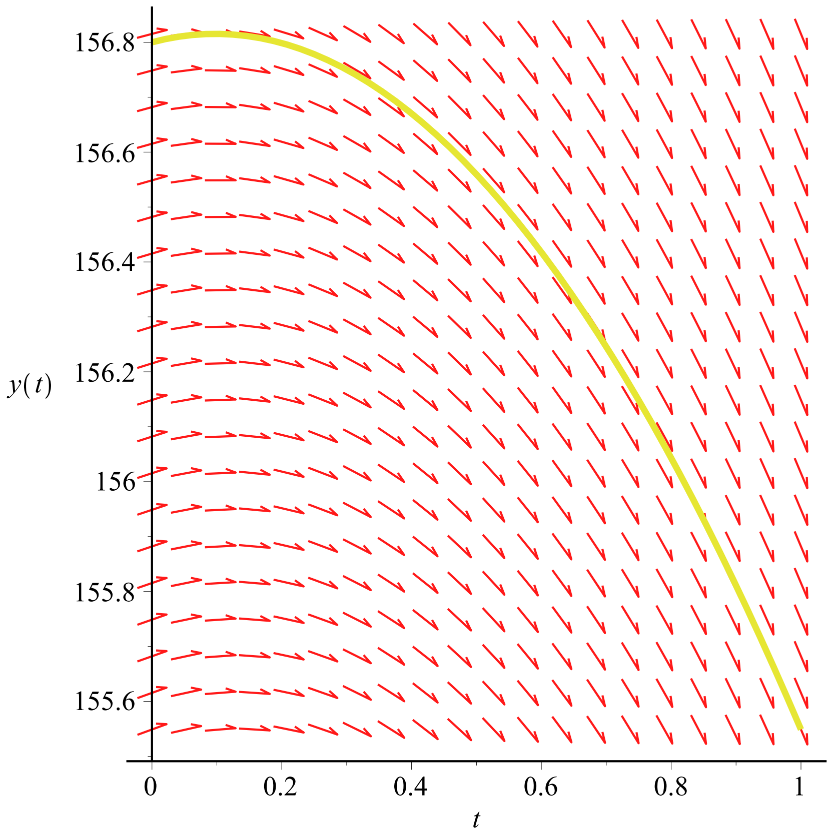


2) This is Q(0)=160-3.2 so...

$160 - 3.2$

$$156.8 \qquad \textbf{(20)}$$

3) $DEplot\left(diff(y(t), t) = .1 \cdot y(t) \cdot \left(1 - \frac{y(t)}{160}\right) - 3.2 \cdot t, y(t), t = 0..1, [[y(0) = 156.8]]\right)$



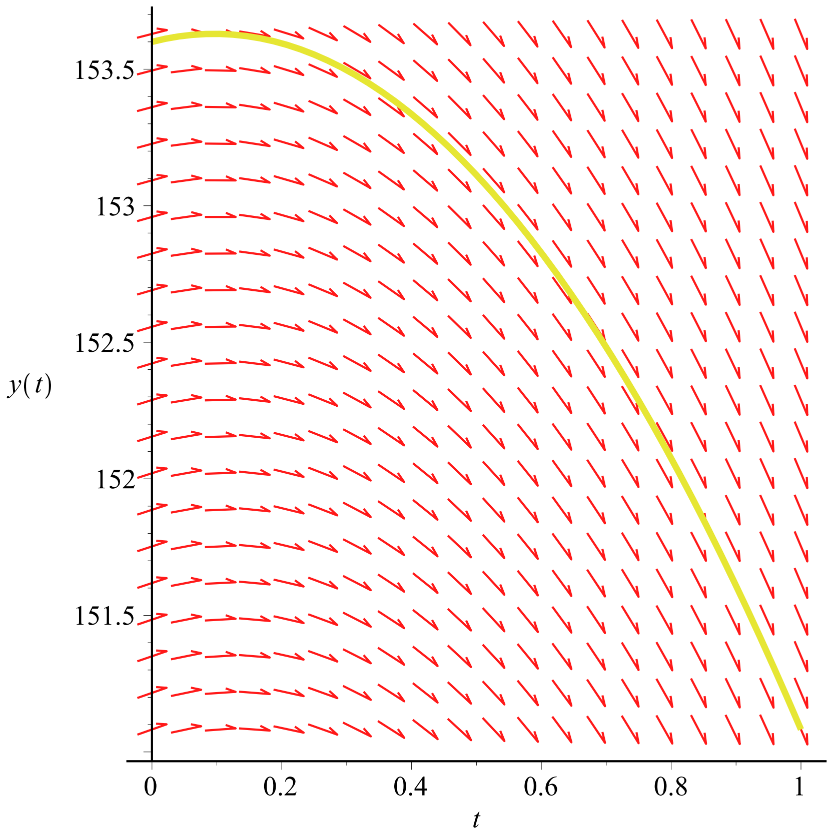
4) It regenerates faster but will approach a horizontal assumptope at 160, the carrying capacity.

5) $\frac{dQ}{dt} = .1 \cdot Q\left(1 - \frac{Q - 6.4 \cdot t}{160}\right)$ with Q(0)=160-6.4 or 153.6

$160 - 6.4$

$$153.6 \tag{21}$$

$DEplot\left(diff\left(y(t), t\right) = .1 \cdot y(t) \cdot \left(1 - \frac{y(t)}{160}\right) - 6.4 \cdot t, y(t), t = 0 ..1, [[y(0) = 153.6]]\right)$



C)

Strategy 1

$$\frac{dy}{dt} = \frac{y}{10}\left(1 - \frac{y}{160}\right) - 3.2\,t$$

$$\frac{dy}{dt} = \frac{1}{10}\,y\left(1 - \frac{1}{160}\,y\right) - 3.2\,t \qquad \textbf{(22)}$$

$$0 = \frac{y}{10}\left(1 - \frac{y}{160}\right) - 3.2\,t$$

$$0 = \frac{1}{10}\,y\left(1 - \frac{1}{160}\,y\right) - 3.2\,t \qquad \textbf{(23)}$$

$$t = 0 + \frac{y}{32} - \frac{y^2}{5120}$$

$$t = \frac{1}{32}\,y - \frac{1}{5120}\,y^2 \qquad \textbf{(24)}$$

$$t = \frac{\left(\frac{-1}{32} \pm \sqrt{\left(\frac{1}{32}\right)^2 - 4\left(\frac{-1}{5120}\right)(0)} \right)}{2\left(-\frac{1}{5120}\right)}$$

$$t = 80 - 2560 \pm \frac{3}{160}\sqrt{5} \tag{25}$$

Strategy 2

$$\frac{dy}{dt} = \frac{y}{10}\left(1 - \frac{y}{160}\right) - 6.4\,t$$

$$\frac{dy}{dt} = \frac{1}{10}\,y\left(1 - \frac{1}{160}\,y\right) - 6.4\,t$$

$$0 = \frac{y}{10}\left(1 - \frac{y}{160}\right) - 6.4\,t$$

$$0 = \frac{1}{10}\,y\left(1 - \frac{1}{160}\,y\right) - 6.4\,t$$

$$t = 0 + \frac{y}{64} - \frac{y^2}{10240}$$

$$t = \frac{1}{64}\,y - \frac{1}{10240}\,y^2 \tag{28}$$

$$t = \frac{\left(\frac{-1}{64} \pm \sqrt{\left(\frac{1}{64}\right)^2 - 4\left(\frac{-1}{10240}\right)(0)} \right)}{2\left(-\frac{1}{10240}\right)}$$

$$t = 80 - 5120 \pm \frac{1}{320}\sqrt{65} \tag{29}$$

These are the points on the graph where the slope is 0 and therefore, in the context of the problem means the amount that regrows every year equals the amount that is cut down every year which can be either constant in a .

D)
1) 3.2 is the average yeild.
2) Q(t) = .99(160)
Recovery time is 94 years for this forest. So each cycle is 94 years so quantity harvested is about .99 (160)=158.4 hardwood every 94 years so the average annuel yeild for a cycle is 158.4/94=1.685106383
3) 2 periods in this cycle. Harvesting period and recovery period. Recovery periof is 94 years so recovery period t=94 and quantity harvested = 0. The harvesting period lasts until Q(t)=1.3 so t is...

$$\left(\frac{10\ln(1.3) + \frac{160}{1.3}}{-1.6} \right)^{\frac{1}{2}}$$

$$8.863568897\ \mathrm{I} \tag{30}$$

so the yeild for each of these periods will be 158.7 and each will last about 103 years and so the average wood harvested per year is 158.7/103=1.540776699

E)

1) $\frac{dy}{dt} = \frac{y}{10}\left(1 - \frac{y}{160}\right) - C$ where C is the amount arvested every year and dy/dt = 0

$C = \frac{y}{10}\left(1 - \frac{y}{160}\right)$

2) This is false because sustainable harvesting causes constastant human presence as well as maintaining a carrying capacity which is below the max because the carrying capacity of the forest in then where dy/dt=0 with constant harvesting. Therefore, this does not maintain the virgin state of forests as maintained.

3) This arguement is fundemantally flawed in that if one doesn't reduce harvesting today, harvestable wood will run out or replenish at a slower rate and therefore reduce/eliminate jobs in the future. This can be seen by comparing part A with strategies 1 and 2 in part B.