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Calculus 2

Lab 1

A) See Maple attachment

B)

1. The corresponding method of each integral displays consistent patterns when comparing the size of error with a given n and with n twice as large. The errors for L(10)/L(5), L(20)/L(10), L(40)/L(20), L(80)/L(40), L(160)/L(80), as well as the errors for R(10)/R(5), R(20)/R(10), R(40)/R(20), R(80)/R(40), R(160)/R(80) are all approximately 0.5. That is, there is a 1:2 ratio in size of 2n:n for the left and right sum errors meaning that the 2n for the left and right sums are twice as accurate as the corresponding n. This is the case for all three integrals. The errors for M(10)/M(5), M(20)/M(10), M(40)/M(20), M(80)/M(40), M(160)/M(80), as well as the errors for T(10)/T(5), T(20)/T(10), T(40)/T(20), T(80)/T(40), T(160)/T(80) are all approximately 0.25. Here, there is a 1:4 ratio in size of 2n:n for the middle and trapezoid sum errors meaning that the 2n for the middle and trapezoid sum errors are four times more accurate than the corresponding n. Again, this is the case for all three integrals.

2. When comparing the sizes of the errors for the four different methods on the same integral with the same n, there are a number of noticeable trends. The size of the error of the left and right sums are relatively identical. The only difference between the left and right sum errors is their sign (if left is negative, then right is positive, and vice-versa). This depends on the concavity of the function on the specific interval. With regard to the size of the trapezoid and middle sums, the trapezoid sum error is double the size of the middle sum error. This means that the middle sum is twice as accurate as the trapezoid sum. Similarly to the left and right sums, the trapezoid and middle sums have opposite signs (if trap is negative, then middle is positive, and vice-versa). This is the case in each n of each of the three integrals.

3. The sign of the error for MID(n) is positive when the concavity of y=f(x) on interval [a,b] is concave down. The sign of the error for MID(n) is negative when the concavity of y=f(x) on interval [a,b] is concave up.

C)

1. The size of Simpson’s Rule error for each integral at each of its n is significantly smaller than the size of the rest of the methods’ errors for the respective integrals and n. Also, the sizes of the errors for Simpson’s Rule and other methods for each n = 5, 10, 20, 40, 80, 160 follow a distinct trend. The error for TRAP/SIMP is double the size of the error for MID/SIMP for all n. This means that the middle sum is twice as accurate as the trapezoid sum. The error for LEFT/SIMP and RIGHT/SIMP are relatively the same in size and therefore have similar accuracies. This is true for each of the three integrals.

2. Simpson’s Rule is apparently more accurate because there will be cancellation in the errors in the trapezoid and middle sums. Simpson’s Rule is a weighted averaging of the trapezoid and middle sums (which are the two more accurate approximations to begin with). The negative value (underestimate) of one will be countered by the positive value (overestimate) of the other.