

College of the Holy Cross, Spring 2014
Math 136 – Midterm Exam 3
May 2

Your Name: _____

Instructions: For full credit, you must show *all work* on the test pages and place your final answer in the box provided for the problem. Use the back of the preceding page if you need more space for scratch work. The numbers next to each part of the questions are their point values.

Please do not write in the space below

Problem	Points/Poss
I	/ 25
II	/ 20
III	/ 10
IV	/ 20
V	/ 25
Total	/100

I. Let R be the region bounded by $y = \sin(x)$, the x -axis, and $0 \leq x \leq \pi/3$.

A. [10 points] Write down (but *do not try to evaluate*) the integral that would compute the arc-length of the top edge of R .

Arclength integral:

B. [15 points] A thin metal plate of constant density has the shape of R . Find the x -coordinate of the center of mass of the plate.

$\bar{x} =$

II. Both parts of this problem deal with the differential equation $y' = xy$.

- A. [15 points] Find the general solution $y(x)$ of the equation by separating variables and integrating.

- B. [5 points] Find the particular solution $y(x)$ satisfying the initial condition $y(0) = 4$ and compute the exact value of $y(2)$.

Particular solution:

$y(2) =$

III. [10 points] A drug is administered to a patient intravenously at a constant rate of 10mg per hour. The patient's body breaks down the drug and removes it from the bloodstream at a rate proportional to the amount present, with some proportionality constant k . Write a differential equation for the function $Q(t)$ = amount of the drug present (in mg) in the bloodstream at time t (in hours) that describes this situation. Note: You *do not need to solve* the equation.

Differential Equation:

IV.

- A. [10 points] Does the geometric series $\sum_{n=0}^{\infty} \frac{2^n}{\pi^n}$ converge or diverge? If it is convergent, say why and find the sum; if it is not convergent say why not.

Answer:

- B. [10 points] Explain why the Integral Test can be applied to the series $\sum_{n=1}^{\infty} \frac{1}{n^2 + 4}$ and use it to determine if the series converges or diverges.

Answer:

V. All parts of this question refer to the power series

$$\sum_{n=1}^{\infty} \frac{(x-1)^n}{n3^n}.$$

A. [15 points] Use the Ratio Test to determine the radius of convergence.

Radius of convergence:

B. [10 points] Test convergence at the endpoints of the interval from part A to determine the interval of convergence. Explain your conclusions.

Interval of convergence: